

CDGT Dual Program Math Level 1, Part II

Homework 3 : Due Apr 6

1. Find the area between the graph of the function and the x -axis over the given interval.

(a) $y = 3x^2 + e^x$ over the interval $[1, 3]$

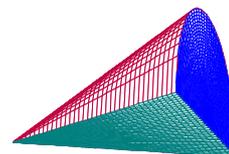
(b) $y = x \cos(x^2)$ over the interval $[0, \pi]$

(c) $y = |x^3| + 3$ over the interval $[-2, 2]$

2. Express the limit as the limit of a suitable Riemann sum, and then compute it by evaluating an appropriate integral.

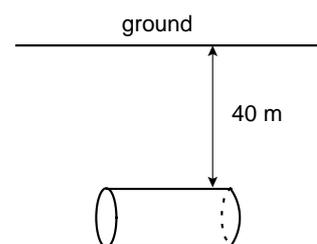
$$\lim_{n \rightarrow \infty} \left(\frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{2n+n} \right)$$

3. The base of a certain solid is an equilateral triangle with altitude 14 inches. Cross-sections perpendicular to the altitude are semicircles. Find the volume of the solid.



4. Consider the finite region on the xy -plane enclosed by the curves $y = x^2 - 2$ and $y = x + 4$.
- (a) Find the volume of the solid obtained by rotating the given region about the x -axis.
- (b) Find the volume of the solid obtained by revolving the given region about the y -axis.

5. A gas station stores its gasoline in a tank under the ground. The tank is a cylinder lying horizontally on its side. If the radius of the cylinder is 5 meters, its length is 20 meters, and its top is 40 meters under the ground, find the total amount of work needed to pump the gasoline out of the tank up to the ground. (The density of gasoline is 673 kg/m^3 ; use $g = 9.8 \text{ m/s}^2$.)



6. To understand the Fundamental Theorem of Calculus: Find the derivative of the area function $A(x) = \int_0^x \cos t \, dt$ by using the limit definition of derivative. How can you recover $A(x)$ from $A'(x)$? Can you find $\frac{d}{dx} \int_0^{x^2} \cos t \, dt$ without integrating the function first?