Torque about a Fixed Axis

- In general, for a rigid body rotating about the z-axis, by

$$\boldsymbol{\tau}_{total}^{ext} = \frac{d\mathbf{L}}{dt}$$

  we have

$$\boldsymbol{\tau}_{total\ z}^{ext} = \frac{d\mathbf{L}_z}{dt}$$

- We shall call $\boldsymbol{\tau}^{ext}{}_{total\,z}$ the torque about the z-axis

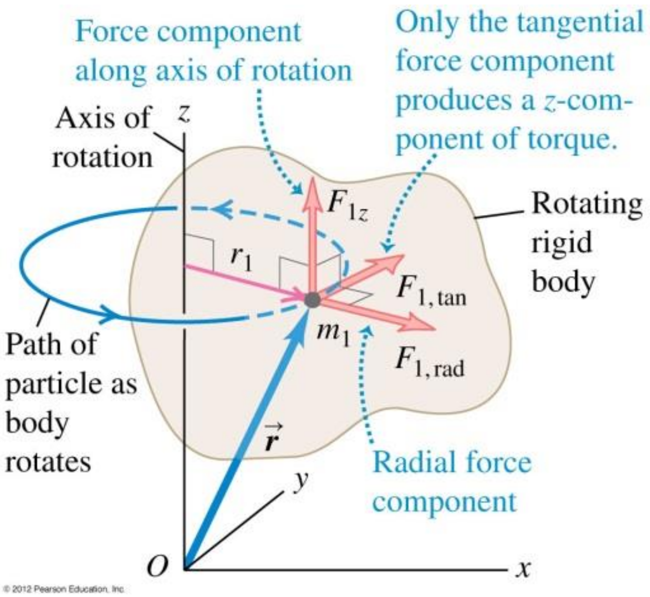
Torque about a Fixed Axis

$$\boldsymbol{\tau}_{total}^{ext} = \sum_{i=1}^{N} \mathbf{r}_i \times \mathbf{F}_i^{ext}$$

$$= \sum_{i=1}^{N} \left(\mathbf{r}_{iz} + \mathbf{r}_{i\perp}\right) \times \left(\mathbf{F}_{iz}^{ext} + \mathbf{F}_{i\mathrm{rad}}^{ext} + \mathbf{F}_{i\tan}^{ext}\right)$$

$$= \sum_{i=1}^{N} \mathbf{r}_{iz} \times \left(\mathbf{F}_{i\mathrm{rad}}^{ext} + \mathbf{F}_{i\tan}^{ext}\right) + \sum_{i=1}^{N} \mathbf{r}_{i\perp} \times \left(\mathbf{F}_{iz}^{ext} + \mathbf{F}_{i\tan}^{ext}\right)$$

$$\boldsymbol{\tau}_{total\ z}^{ext} = \sum_{i=1}^{N} \mathbf{r}_{i\perp} \times \mathbf{F}_{i\tan}^{ext}$$



In other words, to find the torque about the z-axis acting on a rigid body:

1) Consider one small element of the rigid body
2) Find the product of (the perpendicular distance of this element from the axis) and (the magnitude of the tangential external force acting on this element)
3) Determine the sign of this product by right-hand rule
4) Add over all elements of the rigid body

"Newton's Second Law" for Rotation

- For rotation about a fixed axis

$$\tau_{total\ z}^{ext} = \frac{dL_z}{dt} = \frac{d}{dt}\left(I\omega\right)$$

- If the moment of inertia about the axis is a constant, then

$$\tau_{total\ z}^{ext} = I\frac{d\omega}{dt} = I\alpha$$

- If the net torque is zero, the angular acceleration is zero and thus the angular velocity of the body will not be changed

Example: Find the acceleration of the block, angular acceleration of the disk and the tension in the cable. Given M = 2.5kg, R = 20cm, m = 1.2kg, moment of inertia of the disk about a perpendicular axis passing through the center is $I = \frac{1}{2} MR^2$.

Solution:

Let $\hat{\mathbf{n}}$ be the unit vector pointing into the page (clockwise rotation = positive).

Let the angular acceleration of the disk be α, and the linear acceleration of the mass be a (downward = positive). Then

$$a = \alpha R$$

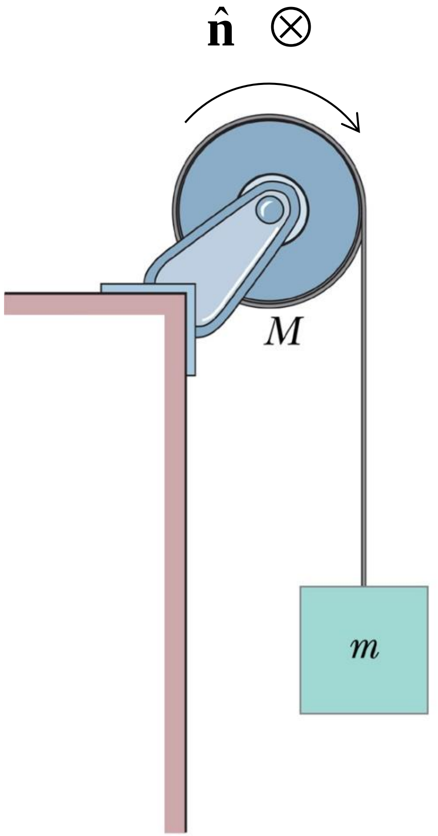
The torque about the center of the disk is

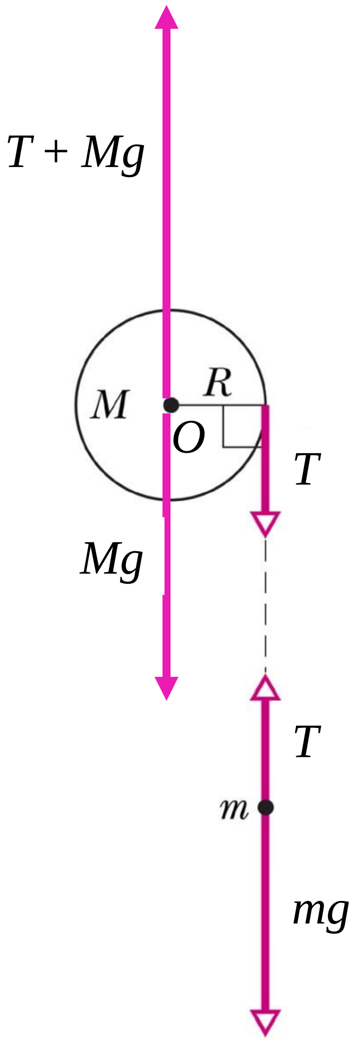
$$\tau = RT$$

Hence $$\alpha = \frac{\tau}{I} = \frac{RT}{MR^2/2} = \frac{2T}{MR}$$

For the mass m, we have

$$mg - T = ma$$





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Solution:

Hence

$$mg - T = m\alpha R = m\frac{2T}{MR}R = \frac{2m}{M}T$$

$$\Rightarrow mg = \left(1 + \frac{2m}{M}\right)T$$

$$\Rightarrow T = \frac{mMg}{2m+M}$$

$$\alpha = \frac{2T}{MR} = \frac{2}{MR}\frac{mMg}{2m+M} = \frac{2mg}{(2m+M)R}$$

$$a = \alpha R = \frac{2m}{2m+M}g$$