Dual Program Level 1 Physics Course

Assignment

Take g = 9.8 ms^{-2} and ignore air resistance, unless otherwise specified.

Moment of inertia of a uniform rod with mass M and length Λ about a perpendicular axis passing through its center: $I = \frac{1}{12} M \Lambda^2$.

Moment of inertia of a uniform disk with mass M and radius R about a perpendicular axis passing through its center: $I = \frac{1}{2} MR^2$.

Moment of inertia of a uniform cylinder with mass M and radius R about its axis of symmetry: $I = \frac{1}{2} MR^2$.

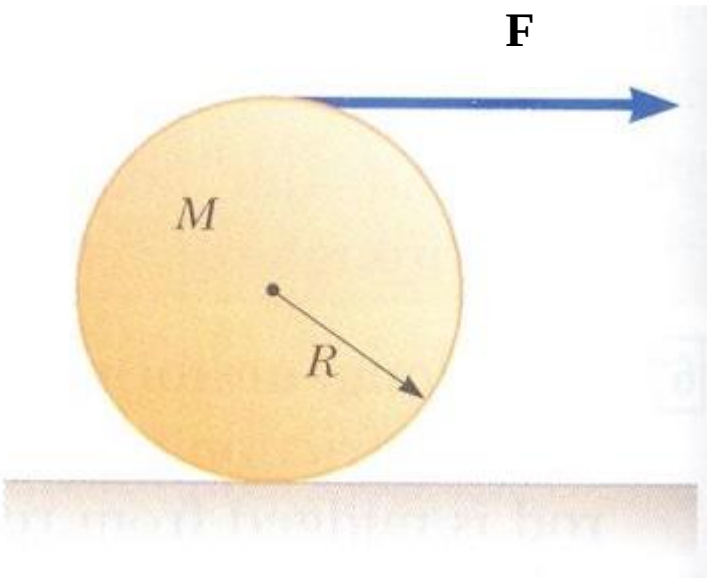
Moment of inertia of a uniform sphere with mass M and radius R about any axis passing through its center: $I = \frac{2}{5} MR^2$.

1. (a) Argue that among the set of parallel axes about which an object can rotate, the object has the smallest moment of inertia about the axis passing through its CM.
   (b) Find the moment of inertia of a uniform thin disk with mass M and radius R about any diameter.

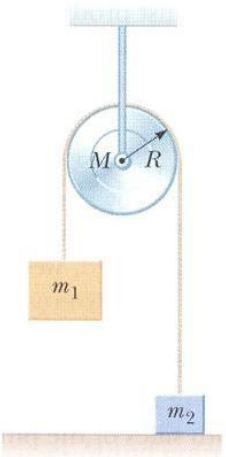
2. Two objects with masses m and $2m$ are attached to the two ends of a uniform rod with length Λ.
   (a) If the rod is a light rod, find the moment of inertia of the system about an axis passing through its CM and perpendicular to the rod.
   (b) If the rod has mass m, find the moment of inertia **of the rod** about the CM of the whole system by
      (i) first principle and integration
      (ii) parallel axis theorem.

      Also obtain the moment of inertia **of the whole system** about its CM.

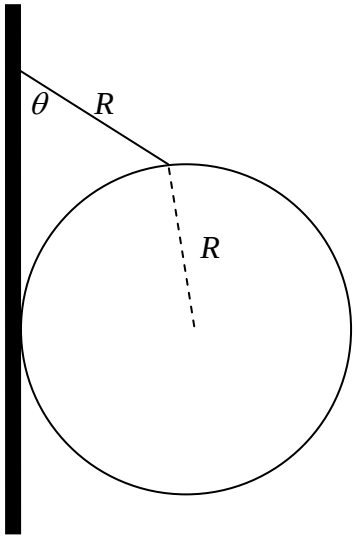
3. A spool of wire of mass M and radius R is unwound under a constant force **F**. Assume that the spool is a uniform solid cylinder that doesn't slip.
   (a) Show that the acceleration of the center of mass is $4\mathbf{F}/3M$.
   (b) Show that the force of friction is to the right and equal in magnitude to $F/3$.
   (c) If the cylinder starts from rest and rolls without slipping, what is the speed of its center of mass after it has rolled through a distance d?



4. Consider the system shown in the figure below, with m_1 = 20 kg, m_2 = 12.5 kg, R = 0.2 m, and the mass of the pulley M = 5 kg. Object m_2 is resting on the floor, and object m_1 is 4 m above the floor when it is released from rest. The pulley axis is frictionless. The cord is light, does not stretch, and does not slip on the pulley.
   (a) Calculate the time interval required for m_1 to hit the floor.
   (b) How would your answer change if the pulley were massless?



5. A string is attached to a point on the surface of a uniform sphere with radius R mass M. The other end is attached to a point on a smooth wall. The length of the string is also R. The sphere is in an equilibrium position as shown in the figure below. Find the normal reaction of the wall, the tension of the string, and the angle θ.



Smooth wall

6. Two identical point objects both with mass m are attached to the two ends of a uniform rod with length Λ. The mass of the rod is also m. The rod is initially at rest on the xy plane, with one end at the origin and the other end on the positive y-axis. A third point object with mass m is initially moving with velocity v on the negative x-axis. At $t = 0$, it hits the end of the rod at the origin and stick to it afterwards.
   (a) What is the motion of the CM of the combined object after the collision?
   (b) What is the angular velocity (magnitude and direction) of the combined object after the collision when observed in the CM frame?

   You can use the result of question 2.