# Dual Program Level 1 Physics Course

## Assignment 7

**Due: 26/Nov/2011 14:00**

1. Imagine that there are only five currencies in the world: HK dollar (HKD), US dollar (USD), Euro (EUR), Japanese Yen (JPY), and a mysterious currency X. The exchange rates are given by:

   1 USD to 7.78890 HKD
   1 USD to 76.9050 JPY
   1 EUR to 10.5349 HKD
   1 X to $10^{24}$ USD

   It is assumed that these rates always remain the same.

   In the foreign currency account of an investor, the amounts of different currencies are $V_{\text{HKD}}$, $V_{\text{USD}}$, $V_{\text{EUR}}$, $V_{\text{JPY}}$, and $V_{\text{X}}$.
   There are two rules in this imaginary financial world:
   (1) The investor can only use other currencies to buy X, but not vice versa.
   (2) In every transaction, 10% of the amount will be used to buy X. For example, to buy USD with 100 HKD, only 90 HKD will be used to buy USD, the remaining 10 HKD will be used to buy X.

   A hacker broke into the investor's computer and gained access to the five numbers $V_{\text{HKD}}$, $V_{\text{USD}}$, $V_{\text{EUR}}$, $V_{\text{JPY}}$, and $V_{\text{X}}$, without knowing their actual meaning. He also failed to notice the small changes in $V_X$.

   (a) Describe what the hacker observed about the values of $V_{\text{HKD}}$, $V_{\text{USD}}$, $V_{\text{EUR}}$, $V_{\text{JPY}}$, assuming that the investor had been doing a lot of transactions on all the different currencies.

   After an international meeting, it was agreed that the currency X be kicked out from the market.

   (b) One day, the hacker observed changes $\Delta V_{\text{HKD}}$ and $\Delta V_{\text{USD}}$ in the account of HKD and USD, respectively, while $V_{\text{JPY}}$ and $V_{\text{EUR}}$ remain unchanged. Here $\Delta V > 0$ ($< 0$) means increase (decrease) in $V$. What is the ratio $|\Delta V_{\text{HKD}}| / |\Delta V_{\text{USD}}|$ he observed?

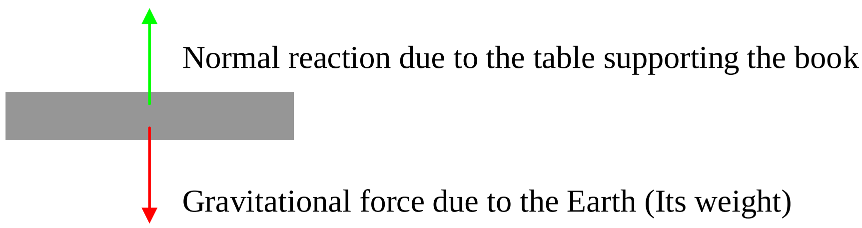
(c) Another day, the hacker observed changes $\Delta V_{\text{EUR}}$ and $\Delta V_{\text{JPY}}$ in the account of EUR and JPY, respectively, while $V_{\text{USD}}$ and $V_{\text{HKD}}$ remain unchanged. What is the ratio $|\Delta V_{\text{EUR}}| / |\Delta V_{\text{JPY}}|$ he observed? State what rule you've used to obtain the result.

After a long time of observations, the hacker obtained a law on $V_{\text{HKD}}$, $V_{\text{USD}}$, $V_{\text{EUR}}$, $V_{\text{JPY}}$.

(d) Describe the law in terms of $\Delta V_{\text{HKD}}$, $\Delta V_{\text{USD}}$, $\Delta V_{\text{EUR}}$, $\Delta V_{\text{JPY}}$, in the form of action-reaction.

(e) Describe the law in a conservation form in terms of $V_{\text{HKD}}$, $V_{\text{USD}}$, $V_{\text{EUR}}$, and $V_{\text{JPY}}$.

2. A book of mass $m$ rests on the table. There are three objects to consider:
(i) the book, (ii) the table, (iii) the Earth.
The "free-body diagram" is a diagram showing all the forces acting on an object. For example, the free-body diagram of the book is:




(a) Draw the free-body diagrams of the other two objects. Label clearly all the forces acting on each one of them due to the other two.

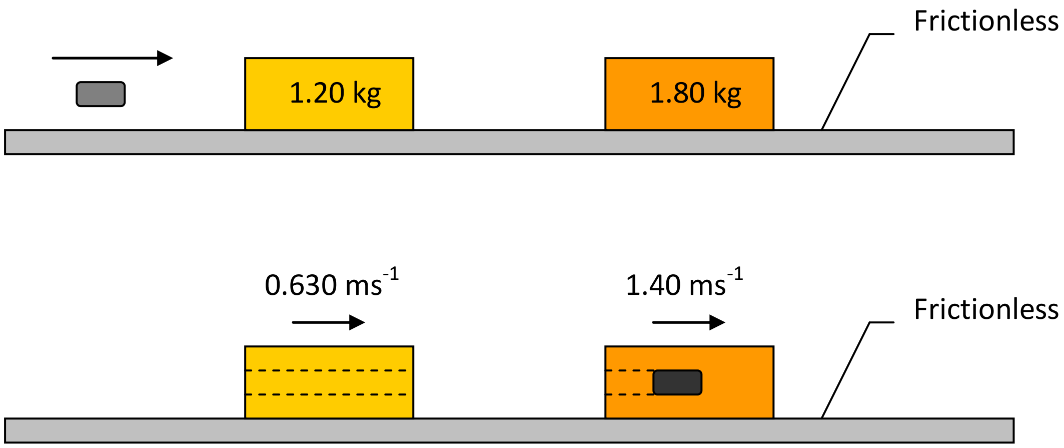
(b) Identify all action-reaction pairs in all the three diagrams.

3. An object $A$ moves to the right with a speed of 5 m/s; another object $B$ moves to the left with a speed of 2 m/s. After a collision, $A$ moves to the left with a speed of 3 m/s; $B$ moves to the right with a speed of 2 m/s.
In another collision experiment, initially $A$ moves to the right with a speed of 22 m/s; a third object $C$ moves to the left with a speed of 1 m/s. After a collision, $A$ moves to the right with a speed of 2 m/s; $C$ also moves to the right with a speed of 3 m/s.
Suppose $B$ is a box and $C$ is a ball. In a third experiment, initially, $B$ moves to the right with a speed of 0.14 m/s. When it passes the observer, he puts $C$ into $B$ so that they move together afterwards. What is their final speed?

4. As shown in the figure below, a 3.50 g bullet is fired horizontally at two blocks at rest on a frictionless tabletop. The bullet passes through the first block, with mass 1.20 kg, and embeds itself in the second, with mass 1.80 kg. Speeds of 0.630 $ms^{-1}$ and 1.40 $ms^{-1}$, respectively, are thereby given to the blocks. Neglecting the mass removed from the first block by the bullet, find

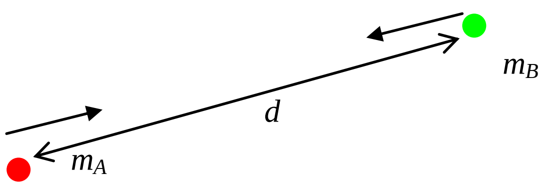
(a) the speed of the bullet immediately after it emerges from the first block and

(b) the bullet's original speed.




5. After a collision, two objects of the same mass and same initial speed are found to move away together at 1/2 their initial speed. Find the angle between the initial velocities of the objects.

6. Two objects, $A$ and $B$, initially at rest, are separated by a distance $d$. They move towards each other due to the mutual attraction and then collide.
The distance of the point of collision from the initial position of $A$ and $B$ are $r_A$ and $r_B$, respectively. Find $r_A$ and $r_B$ in terms of $m_A$ (the mass of $A$), $m_B$ (the mass of $B$) and $d$. What is the ratio $r_A/r_B$?




7. (a) What is the SI unit of angular momentum?

(b) At a certain moment, ship A is at 1 km due east of a pier, and moving due west with a speed of 5 m/s. Find the angular momentum of the ship (direction and magnitude) with respect to the pier.

(c) Unlike linear momentum, angular momentum depends on the choice of reference point from which **r** is measured. Repeat part (b), but with reference point *O*’ which is at a distance of 2 km to the pier in the north direction.

8. Assume

(I) The Earth and the Moon are performing circular motions around a common center, called the center of mass (CM), which is at rest.

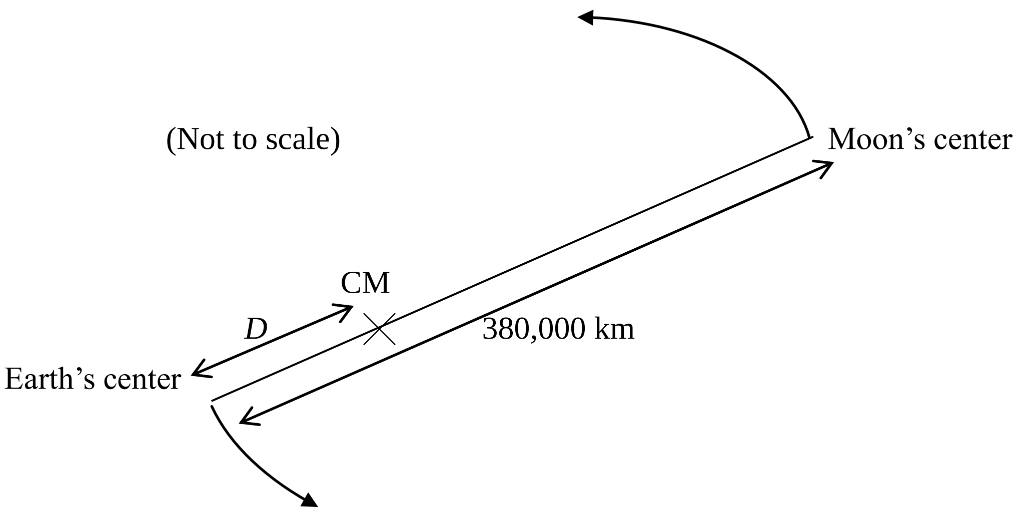
(II) The Earth-Moon distance is 380,000 km.

(III) The period is exactly 27 days 8 hours.

(IV) Both of the Earth and the Moon can be considered as point mass.

The centripetal forces are provided by the gravitational attractions between the Earth and the Moon, which are a pair of action-reaction that act along the line joining them.

Mass of the Earth: $m_E = 6.0 \times 10^{24}\,\text{kg}$. Mass of the Moon: $m_M = 7.3 \times 10^{22}\,\text{kg}$.




(a) Argue that the Earth and the Moon must always be on exactly the opposite sides of the CM.

(b) By Newton’s third law, find the distance of the CM from the Earth’s center. Is it inside or outside the Earth? (The radius of the Earth is 6370 km).

(c) Find the orbital speeds of the Earth and the Moon and check that the total linear momentum is conserved.

(d) Find the accelerations of the Earth and the Moon falling towards each other.

(e) Find, with respect to the CM,

(i) the angular momentum of the Earth,

(ii) the angular momentum of the Moon, and

(iii) the total angular momentum of the Earth-Moon system.

Remember to specify both the magnitudes and the directions of the angular momenta.

9. **<u>Optional</u>**

The Tsiolkovsky rocket equation (or ideal rocket equation) is named after the Russian rocket scientist Konstantin Eduardovich Tsiolkovsky, who published it in 1903. The equation gives the change in speed of a rocket when there are no external forces (e.g., gravity, air resistance, etc.).

Consider a rocket with mass $m_i$ in outer-space far from any gravitational sources. The rocket is originally travelling at a constant velocity $v_i$. The astronauts turn on the engine, which is designed to eject gas at a fixed exhaust velocity of $u$ (relative to the rocket). The mass of the rocket decreases when gas is ejected. Show that when its mass decreases to $m_f$, the final velocity $v_f$ is given by

$$v_f - v_i = u \ln \frac{m_i}{m_f} .$$

The function ln is called the natural logarithm, which satisfies

$$\frac{d}{dx} \ln x = \frac{1}{x} \quad \Leftrightarrow \quad \int \frac{1}{x} dx = \ln x + C .$$