

Mathematics (Level 1)

Dr. CHENG, Kam Hang Henry

The Center for the Development of the Gifted and Talented,
The Hong Kong University of Science and Technology



Course introduction

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Main topics in this course

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We will study behaviors of different functions in various aspects.

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- What happens if you key in “ $1 \div 0.01$ ”, “ $1 \div 0.00001$ ”, and $1 \div$ a positive number that is closer and closer to 0?
- What if you key in “ $1 \div -0.01$ ”, “ $1 \div -0.00001$ ”, and $1 \div$ a negative number that is closer and closer to 0?

What is actually happening here?

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- Without using a calculator, how do you find a solution to a **complicated equation** such as

$$x = \cos x?$$

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- How to measure the area of **regions of arbitrary shapes**?
- Why is the **area** of a circular disk of radius r given by

$$A = \pi r^2?$$

Why is the **volume** of a spherical ball of radius r given by

$$V = \frac{4\pi}{3} r^3?$$

How do we compute the **volume**, **surface area**, etc. of other geometric shapes?