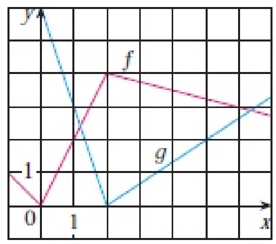
Dual Program 2012 - Mathematics Level 1 — Homework 3

Due: 12-Jan-2013.

1. At what values of x is the following function g differentiable?

$$g(x) = \begin{cases} 2x & \text{if } x \le 0 \\ 2x - x^2 & \text{if } 0 < x < 2 \\ 2 - x & \text{if } x \ge 2 \end{cases}$$

2. Suppose that f and g are functions whose graphs are as follows:



Let:

$$u(x) = f(g(x)), \quad v(x) = g(f(x)), \quad w(x) = g(g(x)).$$

Find each derivative, if it exists. If it does not exist, explain briefly why.

$$\text{(i) } u'(1) \qquad \text{(ii) } v'(1) \qquad \text{(iii) } w'(1).$$

3. The functions f and g are differentiable everywhere, with the following values known:

x	-2	-1	0	1	2
$f(x)$	1	-4	-6	1	-2
$f'(x)$	2	-5	1	3	5
$g(x)$	-1	1	3	6	10
$g'(x)$	-5	-2	1	3	5

Find (i) $\left.\dfrac{d}{dx}\left(f(x) + xg(x)\right)\right|_{x=1}$ (ii) $\left.\dfrac{d}{dx}\left(\ln[(f(x))^2 + 1]\right)\right|_{x=0}$ (iii) $\left.\dfrac{d}{dx}\left(\dfrac{x^2 f(x)}{g(x)}\right)\right|_{x=2}$.

4. Find an equation of the tangent line to the curve at the given point.

   (i) $x^2 + y^2 = (2x^2 + 2y^2 - x)^2$ at $(0, \frac{1}{2})$.

   (ii) $y^2(y^2 - 4) = x^2(x^2 - 5)$ at $(0, -2)$.

5. A balloon is rising at a constant speed of 5 ft/sec. A boy is cycling along a straight road at a speed of 15 ft/sec. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 sec later?

6. A container in the shape of an inverted cone has height 16 cm and radius 5 cm at the top. It is partially filled with liquid that oozes through the sides at a rate proportional to the area of the container that is in contact with the liquid. If we pour the liquid into the container at a rate of 2 cm^3/min, then the height of the liquid decreases at a rate of 0.3 cm/min when the height is 10 cm.

   If we want to keep the liquid at a constant height of 10 cm, at what rate should we pour the liquid into the container?

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