

Textbook Chapter 13 & 14 (Ch. 13. 1 – 13.5, 14.1 – 14.3)

Outline:

1. Temperature
2. Thermal Expansion
3. Heat Capacity & Latent Heat
4. Conduction
5. Ideal Gas Law & Kinetic Theory of Ideal Gas

## What is temperature?

Temperature is a physical property of matter which quantifies the internal state of a matter consisted of numerous particles/molecules. The higher the temperature, the higher the energy per individual particle/molecule has. Temperature is often measured and represented by the macroscopic physical properties of a material, such as the height (volume) of the liquid mercury in a glass tube. When two systems are put in **thermal contact**, after a while, their macroscopic properties become stable and unchanged (e.g. the mercury in a tube stops expanding after certain time sitting in a cup of water.). Then the two systems are said to be in **thermal equilibrium**. In turn, we say they have the same temperature.

SI Unit of temperature: kelvin (K)

Different types of thermometers (each exhibits a different physical property)

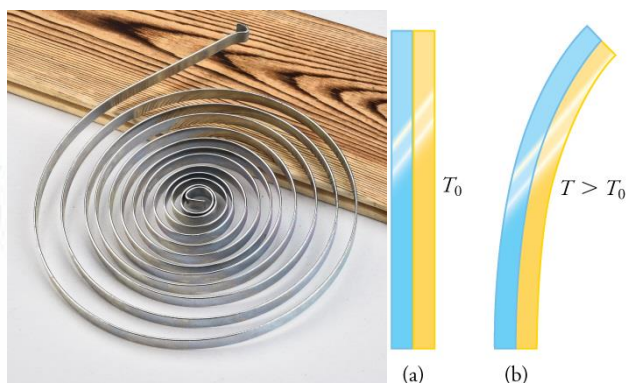


Volume:



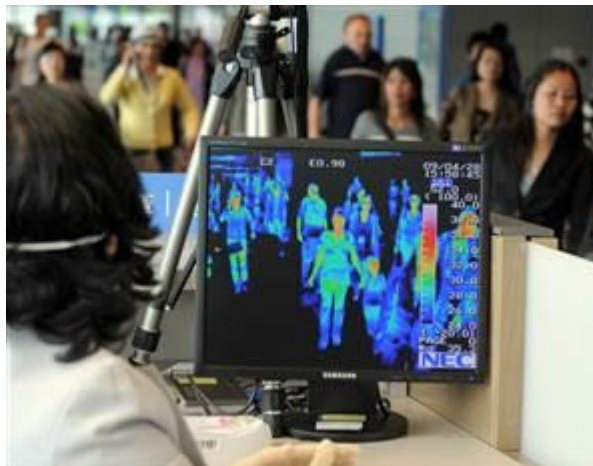
Length:

(Bimetallic thermometer)





Electrical resistance:



Radiation:

### Zeroth Law of Thermodynamics

*If bodies A and B are each in thermal equilibrium with a third body C, then A and B are in thermal equilibrium with each other.*

If body C is a thermometer which gives a temperature reading, then we conclude:

*The temperature of A and B are the same.*

### Temperature Scales

Fahrenheit ( $T_F$ )	Celsius ( $T_C$ )	Kelvin (T)
At 1 atm pressure, freezing point and boiling point of water are $32^\circ$ and $212^\circ$	At 1 atm pressure, freezing point and boiling point of water are $0^\circ$ and $100^\circ$	Lowest possible temperature = 0 K Triple point of water = 273.16 K
$T_F = \frac{9}{5}T_C + 32^\circ$		$T_C = T - 273.15^\circ$

The size of  $1^\circ\text{C}$  is the same as 1 kelvin.  $\rightarrow$   $1^\circ\text{C} = 1\text{ K}$

Notation: The units of different scale are written as  $0^\circ\text{C} = 32^\circ\text{F} = 273.15\text{K}$ .

**Triple point of water**  $T_3$  is the temperature at which all three states of water: gas (vapor), liquid and solid (ice) coexist at pressure of  $0.2\text{ mPa}^*$ . In kelvin, the triple point temperature is

$$T_3 = 273.16 \text{ K}$$

\* mPa is milli-pascal =  $10^{-3}$  Pa

### Some Corresponding Temperatures

Temperature	°C	°F
Boiling point of water <sup>a</sup>	100	212
Normal body temperature	37.0	98.6
Accepted comfort level	20	68
Freezing point of water <sup>a</sup>	0	32
Zero of Fahrenheit scale	≈ -18	0
Scales coincide	-40	-40

<sup>a</sup>Strictly, the boiling point of water on the Celsius scale is 99.975°C, and the freezing point is 0.00°C. Thus, there is slightly less than 100 C° between those two points.



### Thermal Expansion

Linear Expansion:  $\Delta L = \alpha L \Delta T$

Volume Expansion:  $\Delta V = \beta V \Delta T$

If all dimensions of a solid expand with temperature, the volume of that solid must also expand. For liquids, volume expansion is the only meaningful expansion parameter.

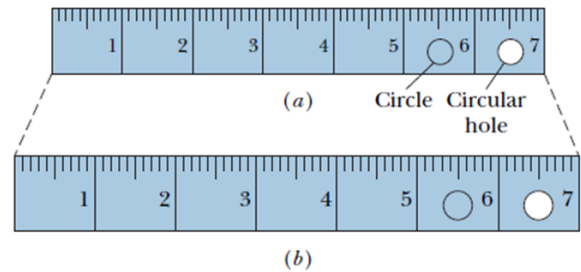
If the temperature of a solid or liquid whose volume is  $V$  is increased by an amount  $\Delta T$ , the increase in volume is found to be

$$\Delta V = V\beta\Delta T$$

where  $\beta$  is the **coefficient of volume expansion of the solid or liquid**. The coefficients of volume expansion and linear expansion for a solid are related by

$$\beta = 3\alpha$$

It is because, suppose  $V = L^3$ ,  $dV = 3L^2 dL = 3L^2(\alpha L dT) = 3\alpha V dT$ .  $\rightarrow \beta = 3\alpha$



**Fig. 18-11** The same steel ruler at two different temperatures. When it expands, the scale, the numbers, the thickness, and the diameters of the circle and circular hole are all increased by the same factor. (The expansion has been exaggerated for clarity.)

Table 13.2 Thermal Expansion Coefficients at 20°C<sup>[1]</sup>

Material	Coefficient of linear expansion $\alpha(1/^\circ\text{C})$	Coefficient of volume expansion $\beta(1/^\circ\text{C})$
<b>Solids</b>		
Aluminum	$25 \times 10^{-6}$	$75 \times 10^{-6}$
Brass	$19 \times 10^{-6}$	$56 \times 10^{-6}$
Copper	$17 \times 10^{-6}$	$51 \times 10^{-6}$
Gold	$14 \times 10^{-6}$	$42 \times 10^{-6}$
Iron or Steel	$12 \times 10^{-6}$	$35 \times 10^{-6}$
Invar (Nickel-iron alloy)	$0.9 \times 10^{-6}$	$2.7 \times 10^{-6}$
Lead	$29 \times 10^{-6}$	$87 \times 10^{-6}$
Silver	$18 \times 10^{-6}$	$54 \times 10^{-6}$
Glass (ordinary)	$9 \times 10^{-6}$	$27 \times 10^{-6}$
Glass (Pyrex®)	$3 \times 10^{-6}$	$9 \times 10^{-6}$
Quartz	$0.4 \times 10^{-6}$	$1 \times 10^{-6}$
Concrete, Brick	$\sim 12 \times 10^{-6}$	$\sim 36 \times 10^{-6}$
Marble (average)	$7 \times 10^{-6}$	$2.1 \times 10^{-5}$
<b>Liquids</b>		
Ether		$1650 \times 10^{-6}$
Ethyl alcohol		$1100 \times 10^{-6}$
Petrol		$950 \times 10^{-6}$
Glycerin		$500 \times 10^{-6}$
Mercury		$180 \times 10^{-6}$
Water		$210 \times 10^{-6}$
<b>Gases</b>		
Air and most other gases at atmospheric pressure		$3400 \times 10^{-6}$

### Example 13.3 Calculating Linear Thermal Expansion: The Golden Gate Bridge

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from  $-15^\circ\text{C}$  to  $40^\circ\text{C}$ . What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

#### Strategy

Use the equation for linear thermal expansion  $\Delta L = \alpha L \Delta T$  to calculate the change in length,  $\Delta L$ . Use the coefficient of linear expansion,  $\alpha$ , for steel from Table 13.2, and note that the change in temperature,  $\Delta T$ , is  $55^\circ\text{C}$ .

#### Solution

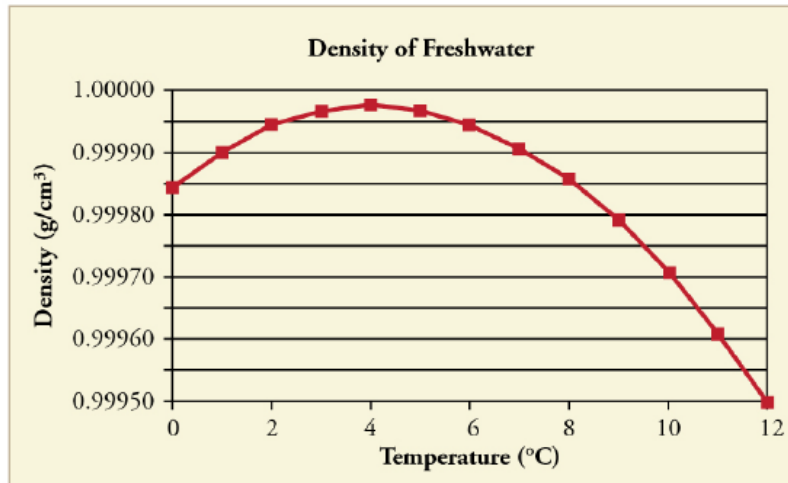
Plug all of the known values into the equation to solve for  $\Delta L$ .

$$\Delta L = \alpha L \Delta T = \left( \frac{12 \times 10^{-6}}{^{\circ}\text{C}} \right) (1275 \text{ m})(55^{\circ}\text{C}) = 0.84 \text{ m.} \quad (13.8)$$

**Discussion**

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

Exception in water:

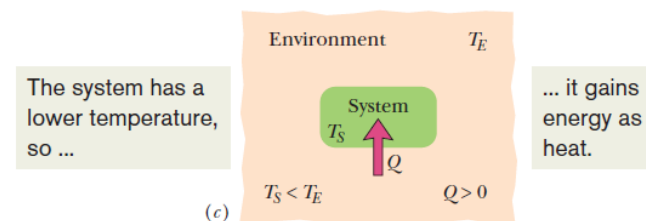
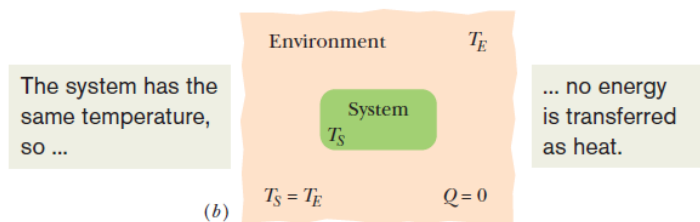
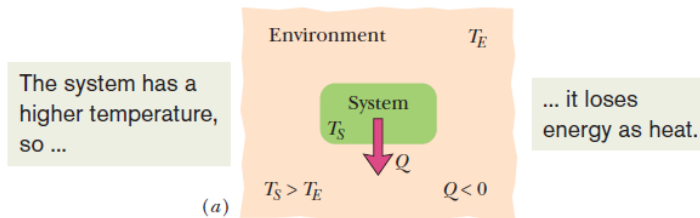


**Figure 13.12** The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at +4°C is only 0.0075% greater than the density at 2°C, and 0.012% greater than that at 0°C.

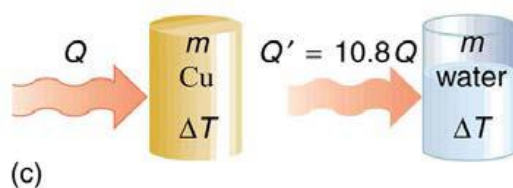
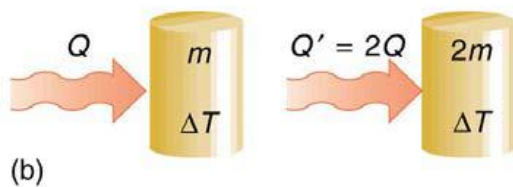
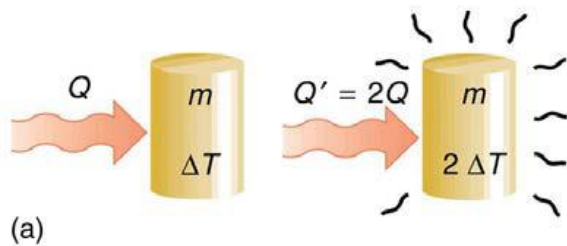
## Heat Transfer

Heat is the energy transfer between from one system to another due to temperature difference.

Suppose one of the system is the environment at temperature  $T_E$



If the temperature of a system exceeds that of its environment as in (a), heat  $Q$  is lost by the system to the environment until thermal equilibrium (b) is established. (c) If the temperature of the system is below that of the environment, heat is absorbed by the system until thermal equilibrium is established.



The heat  $Q$  transferred to cause a temperature change,  $\Delta T$ , depends on the magnitude of the temperature change, the mass of the system, and the substance and phase involved.

(a) The amount of heat transferred is directly proportional to the temperature change. To double the temperature change of a mass  $m$ , you need to add twice the heat.

(b) The amount of heat transferred is also directly proportional to the mass. To cause an equivalent temperature change in a doubled mass, you need to add twice the heat.

(c) The amount of heat transferred depends on the substance and its phase. If it takes an amount  $Q$  of heat to cause a temperature change  $\Delta T$  in a given mass of copper, it will take 10.8 times that amount of heat to cause the equivalent temperature change in the same mass of water assuming no phase change in either substance.

## Heat capacity

The **heat capacity**  $C$  of an object is the proportionality constant between the heat  $Q$  that the object absorbs or loses and the resulting temperature change  $T$  of the object

$$Q = C \Delta T = C (T_f - T_i)$$

The SI unit of  $C$  is J/K.

The value of  $C$  depends on the size of the system, the materials making the system and even the state of the system.

For a system made of one single material, one defines specific heat, the heat capacity per unit mass, for the material. The specific heat  $c$  is related to heat capacity as

$$c = C/m$$

where  $m$  is the mass of the system/substance. So the heat transfer in terms of specific heat is

$$Q = c m \Delta T$$

When quantities are expressed in moles instead of mass, the molar specific heat, heat capacity per mole, is often used.

Some Specific Heats and Molar Specific Heats at Room Temperature

Substance	Specific Heat		Molar Specific Heat
	$\frac{\text{cal}}{\text{g} \cdot \text{K}}$	$\frac{\text{J}}{\text{kg} \cdot \text{K}}$	$\frac{\text{J}}{\text{mol} \cdot \text{K}}$
<i>Elemental Solids</i>			
Lead	0.0305	128	26.5
Tungsten	0.0321	134	24.8
Silver	0.0564	236	25.5
Copper	0.0923	386	24.5
Aluminum	0.215	900	24.4
<i>Other Solids</i>			
Brass	0.092	380	
Granite	0.19	790	
Glass	0.20	840	
Ice ( $-10^\circ\text{C}$ )	0.530	2220	
<i>Liquids</i>			
Mercury	0.033	140	
Ethyl alcohol	0.58	2430	
Seawater	0.93	3900	
Water	1.00	4187	

## Example: Specific Heat

### Example 14.3 Calculating the Final Temperature When Heat Is Transferred Between Two Bodies: Pouring Cold Water in a Hot Pan

Suppose you pour 0.250 kg of  $20.0^\circ\text{C}$  water (about a cup) into a 0.500-kg aluminum pan off the stove with a temperature of  $150^\circ\text{C}$ . Assume that the pan is placed on an insulated pad and that a negligible amount of water boils off. What is the temperature when the water and pan reach thermal equilibrium a short time later?

#### Strategy

The pan is placed on an insulated pad so that little heat transfer occurs with the surroundings. Originally the pan and water are not in thermal equilibrium: the pan is at a higher temperature than the water. Heat transfer then restores thermal equilibrium once the water and pan are in contact. Because heat transfer between the pan and water takes place rapidly, the mass of evaporated water is negligible and the magnitude of the heat lost by the pan is equal to the heat gained by the water. The exchange of heat stops once a thermal equilibrium between the pan and the water is achieved. The heat exchange can be written as  $|Q_{\text{hot}}| = Q_{\text{cold}}$ .

- The values for solids and liquids are at constant volume and at  $25^\circ\text{C}$ , except as noted.
- These values are identical in units of  $\text{cal/g}\cdot^\circ\text{C}$ .
- $c_v$  at constant volume and at  $20.0^\circ\text{C}$ , except as noted, and at 1.00 atm average pressure. Values in parentheses are  $c_p$  at a constant pressure of 1.00 atm.

### Solution

- Use the equation for heat transfer  $Q = mc\Delta T$  to express the heat lost by the aluminum pan in terms of the mass of the pan, the specific heat of aluminum, the initial temperature of the pan, and the final temperature:

$$Q_{\text{hot}} = m_{\text{Al}}c_{\text{Al}}(T_f - 150^\circ\text{C}). \quad (14.12)$$

- Express the heat gained by the water in terms of the mass of the water, the specific heat of water, the initial temperature of the water and the final temperature:

$$Q_{\text{cold}} = m_{\text{W}}c_{\text{W}}(T_f - 20.0^\circ\text{C}). \quad (14.13)$$

- Note that  $Q_{\text{hot}} < 0$  and  $Q_{\text{cold}} > 0$  and that they must sum to zero because the heat lost by the hot pan must be the same as the heat gained by the cold water:

$$Q_{\text{cold}} + Q_{\text{hot}} = 0, \quad (14.14)$$

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$m_{\text{W}}c_{\text{W}}(T_f - 20.0^\circ\text{C}) = -m_{\text{Al}}c_{\text{Al}}(T_f - 150^\circ\text{C}).$$

- This an equation for the unknown final temperature,  $T_f$
- Bring all terms involving  $T_f$  on the left hand side and all other terms on the right hand side. Solve for  $T_f$ ,

$$T_f = \frac{m_{\text{Al}}c_{\text{Al}}(150^\circ\text{C}) + m_{\text{W}}c_{\text{W}}(20.0^\circ\text{C})}{m_{\text{Al}}c_{\text{Al}} + m_{\text{W}}c_{\text{W}}}, \quad (14.15)$$

and insert the numerical values:

$$\begin{aligned} T_f &= \frac{(0.500 \text{ kg})(900 \text{ J/kg}^\circ\text{C})(150^\circ\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})(20.0^\circ\text{C})}{(0.500 \text{ kg})(900 \text{ J/kg}^\circ\text{C}) + (0.250 \text{ kg})(4186 \text{ J/kg}^\circ\text{C})} \\ &= \frac{88430 \text{ J}}{1496.5 \text{ J}^\circ\text{C}} \\ &= 59.1^\circ\text{C}. \end{aligned} \quad (14.16)$$

### Discussion

This is a typical *calorimetry* problem—two bodies at different temperatures are brought in contact with each other and exchange heat until a common temperature is reached. Why is the final temperature so much closer to  $20.0^\circ\text{C}$  than  $150^\circ\text{C}$ ? The reason is that water has a greater specific heat than most common substances and thus undergoes a small temperature change for a given heat transfer. A large body of water, such as a lake, requires a large amount of heat to increase its temperature appreciably. This explains why the temperature of a lake stays relatively constant during a day even when the temperature change of the air is large. However, the water temperature does change over longer times (e.g., summer to winter).

### Check Your Understanding

If 25 kJ is necessary to raise the temperature of a block from  $25^\circ\text{C}$  to  $30^\circ\text{C}$ , how much heat is necessary to heat the block from  $45^\circ\text{C}$  to  $50^\circ\text{C}$ ?

#### Solution

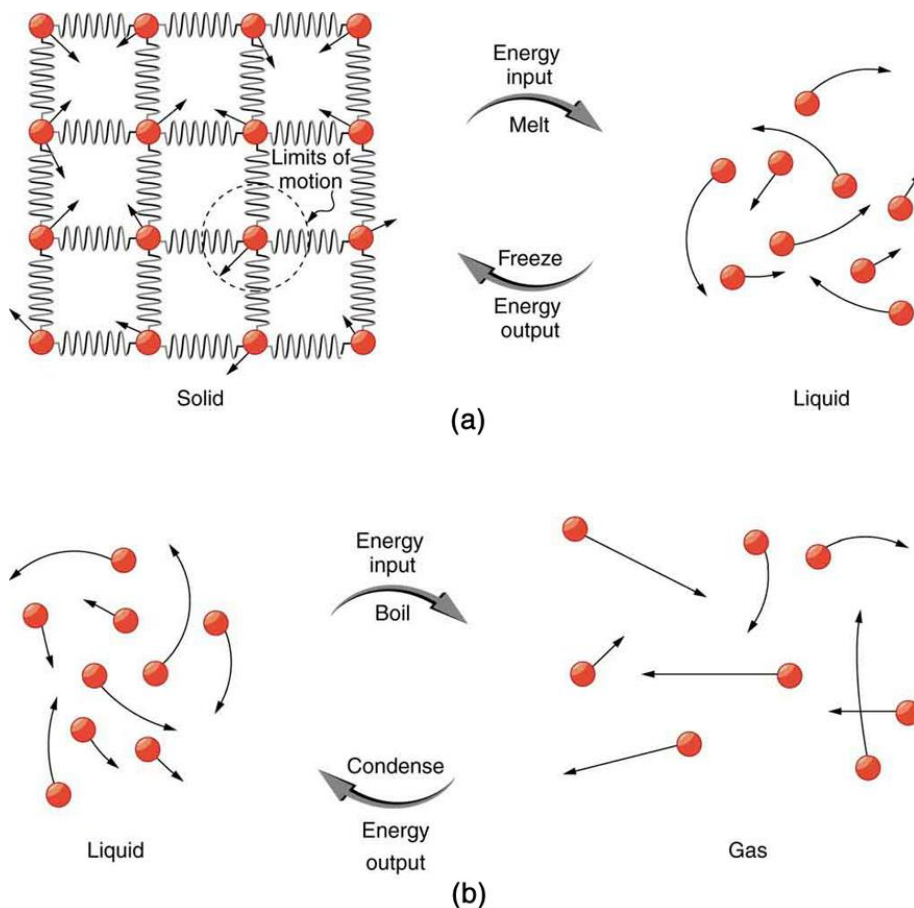
The heat transfer depends only on the temperature difference. Since the temperature differences are the same in both cases, the same 25 kJ is necessary in the second case.



## Latent Heat – Energy for phase change



Heat from the air transfers to the ice causing it to melt.



(a) Energy is required to partially overcome the attractive forces between molecules in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Molecules are separated by large distances when going from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is complete.

The amount of energy per unit mass that must be transferred as heat when a sample completely undergoes a phase change at a fixed temperature is called the **latent heat  $L$** . When a sample of mass

m completely undergoes a phase change, the total energy transferred is:

$$Q = L m$$

When the phase change is between liquid to gas, the heat of transformation is called the **heat of vaporization (or condensation)  $L_V$** .

When the phase change is between solid to liquid, the heat of transformation is called the **heat of fusion  $L_F$  (melting/freezing)**.

Some Heats of Transformation

Substance	Melting		Boiling	
	Melting Point (K)	Heat of Fusion $L_F$ (kJ/kg)	Boiling Point (K)	Heat of Vaporization $L_V$ (kJ/kg)
Hydrogen	14.0	58.0	20.3	455
Oxygen	54.8	13.9	90.2	213
Mercury	234	11.4	630	296
Water	273	333	373	2256
Lead	601	23.2	2017	858
Silver	1235	105	2323	2336
Copper	1356	207	2868	4730

Example 2: Latent Heat

### Example 14.4 Calculate Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Three ice cubes are used to chill a soda at  $20^\circ\text{C}$  with mass  $m_{\text{soda}} = 0.25 \text{ kg}$ . The ice is at  $0^\circ\text{C}$  and each ice cube has a mass of 6.0 g. Assume that the soda is kept in a foam container so that heat loss can be ignored. Assume the soda has the same heat capacity as water. Find the final temperature when all ice has melted.

#### Strategy

The ice cubes are at the melting temperature of  $0^\circ\text{C}$ . Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first the phase change occurs and solid (ice) transforms into liquid water at the melting temperature, then the temperature of this water rises. Melting yields water at  $0^\circ\text{C}$ , so more heat is transferred from the soda to this water until the water plus soda system reaches thermal equilibrium,

$$Q_{\text{ice}} = -Q_{\text{soda}} \quad (14.19)$$

The heat transferred to the ice is  $Q_{\text{ice}} = m_{\text{ice}}L_f + m_{\text{ice}}c_W(T_f - 0^\circ\text{C})$ . The heat given off by the soda is

$Q_{\text{soda}} = m_{\text{soda}}c_W(T_f - 20^\circ\text{C})$ . Since no heat is lost,  $Q_{\text{ice}} = -Q_{\text{soda}}$ , so that

$$m_{\text{ice}}L_f + m_{\text{ice}}c_W(T_f - 0^\circ\text{C}) = -m_{\text{soda}}c_W(T_f - 20^\circ\text{C}). \quad (14.20)$$

Bring all terms involving  $T_f$  on the left-hand-side and all other terms on the right-hand-side. Solve for the unknown quantity  $T_f$ :

$$T_f = \frac{m_{\text{soda}}c_W(20^\circ\text{C}) - m_{\text{ice}}L_f}{(m_{\text{soda}} + m_{\text{ice}})c_W}. \quad (14.21)$$

#### Solution

- Identify the known quantities. The mass of ice is  $m_{\text{ice}} = 3 \times 6.0 \text{ g} = 0.018 \text{ kg}$  and the mass of soda is  $m_{\text{soda}} = 0.25 \text{ kg}$ .

- Calculate the terms in the numerator:

$$m_{\text{soda}}c_W(20^\circ\text{C}) = (0.25 \text{ kg})(4186 \text{ J/kg}\cdot^\circ\text{C})(20^\circ\text{C}) = 20,930 \text{ J} \quad (14.22)$$

and

$$m_{\text{ice}}L_f = (0.018 \text{ kg})(334,000 \text{ J/kg})=6012 \text{ J.} \quad (14.23)$$

3. Calculate the denominator:

$$(m_{\text{soda}} + m_{\text{ice}})c_W = (0.25 \text{ kg} + 0.018 \text{ kg})(4186 \text{ K/(kg}\cdot\text{°C)})=1122 \text{ J/°C.} \quad (14.24)$$

4. Calculate the final temperature:

$$T_f = \frac{20,930 \text{ J} - 6012 \text{ J}}{1122 \text{ J/°C}} = 13\text{°C.} \quad (14.25)$$

### Discussion

This example illustrates the enormous energies involved during a phase change. The mass of ice is about 7 percent the mass of water but leads to a noticeable change in the temperature of soda. Although we assumed that the ice was at the freezing temperature, this is incorrect: the typical temperature is  $-6\text{°C}$ . However, this correction gives a final temperature that is essentially identical to the result we found. Can you explain why?

## Heat Conduction

### Conduction

In the figure on the right, a slab of face area  $A$  and thickness  $L$ , have faces maintained at temperatures  $T_H$  and  $T_C$  by a hot reservoir and a cold reservoir. If  $Q$  be the energy that is transferred as heat through the slab, from its hot face to its cold face, in time  $t$ , then the conduction rate  $P_{\text{cond}}$  (the amount of energy transferred per unit time) is

$$P_{\text{cond}} = \frac{Q}{t} = k A \frac{T_H - T_C}{L}$$

Here  $k$ , called the *thermal conductivity*, is a constant that depends on the material of which the slab is made.

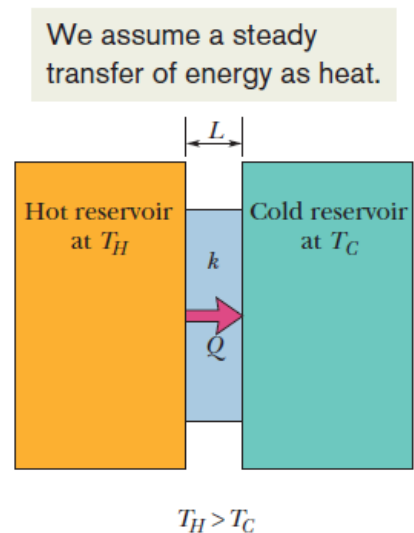
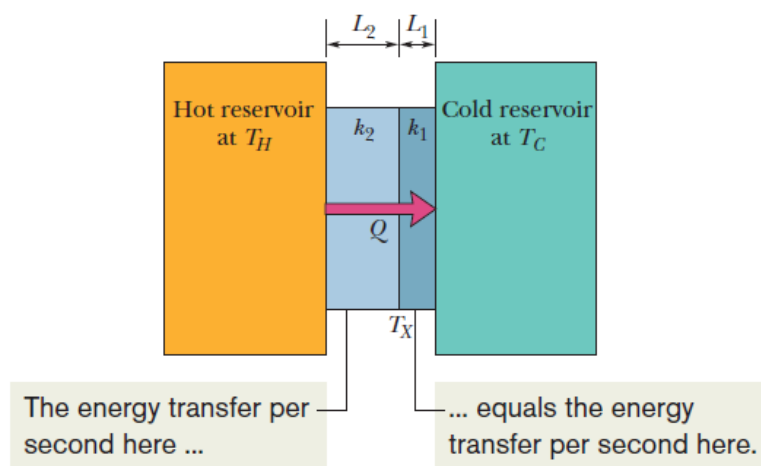


Table 14.3 Thermal Conductivities of Common Substances<sup>[7]</sup>

Substance	Thermal conductivity $k$ (J/s·m·°C)
Silver	420
Copper	390
Gold	318
Aluminum	220
Steel iron	80
Steel (stainless)	14
Ice	2.2
Glass (average)	0.84
Concrete brick	0.84
Water	0.6
Fatty tissue (without blood)	0.2
Asbestos	0.16
Plasterboard	0.16
Wood	0.08–0.16
Snow (dry)	0.10
Cork	0.042
Glass wool	0.042
Wool	0.04
Down feathers	0.025
Air	0.023
Styrofoam	0.010

Example:



Heat is transferred at a steady rate through a composite slab made up of two different materials with different thicknesses and different thermal conductivities. The steady-state temperature at the interface of the two materials is  $T_x$ .

Let  $T_x$  be the temperature of the interface between the two materials. At thermal equilibrium, the

temperature of the interface is steady and the net power entering the interface is zero. So, the power transmitting from the left to interface equal to the power transmitting away from the interface to the right. We have

$$P_{\text{cond}} = \frac{k_2 A (T_H - T_X)}{L_2} = \frac{k_1 A (T_X - T_C)}{L_1}.$$

The temperature of the interface is

$$T_X = \frac{k_1 L_2 T_C + k_2 L_1 T_H}{k_1 L_2 + k_2 L_1}.$$

and the power of conduction is

$$P_{\text{cond}} = \frac{A (T_H - T_C)}{L_1/k_1 + L_2/k_2}.$$

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## Review on Pressure

$$\text{Pressure } P = \frac{\text{Force}}{\text{Area}} = \frac{F}{A}$$

SI Unit: Pascal (Pa),  $1 \text{ Pa} = 1 \text{ N/m}^2 = 1 \text{ kg/m/s}^2$

Atmospheric pressure = 1 atm = 101325 Pa

## Macroscopic variables and Equation of State

The state of a substance consisting of more than  $10^{23}$  particles or molecules is specified by the macroscopic variables: *temperature*  $T$ , *volume*  $V$  and *pressure*  $P$ , instead of the position and velocity of each individual constituent.

When the substance is in thermal equilibrium, the macroscopic variables are related by an **equation of state**  $f(T, V, P) = 0$ . As a result, only two variables are independent.

## Ideal Gas

Ideal gas is an approximated model for real gas. The assumption of ideal gas is that, except elastic collision, there is not any interaction among the gas molecules. A realization of ideal gas is inert gas at low density.

Ideal gas equation of state:  $PV = nRT = NkT$

$n$  is the number of moles.  $N$  is total number of molecules  $N = nN_A$  (Avogadro number  $N_A = 6.022 \times 10^{23}$ .)

$R = 8.31 \text{ J/mol/K}$  is the universal gas constant and  $k = 1.38 \times 10^{-23} \text{ J/K}$  is the Boltzmann constant. They are related by  $R = N_A k$ .

For a fixed amount of gas, the combination of  $\frac{PV}{T} = nR$  is a constant.

### Example 13.6 Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of  $7.00 \times 10^5 \text{ Pa}$  (a gauge pressure of just under  $90.0 \text{ lb/in}^2$ ) at a temperature of  $18.0^\circ\text{C}$ . What is the pressure after its temperature has risen to  $35.0^\circ\text{C}$ ? Assume that there are no appreciable leaks or changes in volume.

#### Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure  $P_0 = 7.00 \times 10^5 \text{ Pa}$ , the initial temperature  $T_0 = 18.0^\circ\text{C}$ , and the final temperature  $T_f = 35.0^\circ\text{C}$ . We must find the final pressure  $P_f$ . How can we use the equation  $PV = NkT$ ? At first, it may seem that not enough information is given, because the volume  $V$  and number of atoms  $N$  are not specified. What we can do is use the equation twice:  $P_0V_0 = NkT_0$  and  $P_fV_f = NkT_f$ . If we divide  $P_fV_f$  by  $P_0V_0$  we can come up with an equation that allows us to solve for  $P_f$ .

$$\frac{P_f V_f}{P_0 V_0} = \frac{N_f k T_f}{N_0 k T_0} \quad (13.20)$$

Since the volume is constant,  $V_f$  and  $V_0$  are the same and they cancel out. The same is true for  $N_f$  and  $N_0$ , and  $k$ , which is a constant. Therefore,

$$\frac{P_f}{P_0} = \frac{T_f}{T_0} \quad (13.21)$$

We can then rearrange this to solve for  $P_f$  :

$$P_f = P_0 \frac{T_f}{T_0} \quad (13.22)$$

where the temperature must be in units of kelvins, because  $T_0$  and  $T_f$  are absolute temperatures.

**Solution**

- Convert temperatures from Celsius to Kelvin.

$$T_0 = (18.0 + 273)\text{K} = 291 \text{ K} \quad (13.23)$$

$$T_f = (35.0 + 273)\text{K} = 308 \text{ K}$$

- Substitute the known values into the equation.

$$P_f = P_0 \frac{T_f}{T_0} = 7.00 \times 10^5 \text{ Pa} \left( \frac{308 \text{ K}}{291 \text{ K}} \right) = 7.41 \times 10^5 \text{ Pa} \quad (13.24)$$

### Example 13.9 Calculating Number of Moles: Gas in a Bike Tire

How many moles of gas are in a bike tire with a volume of  $2.00 \times 10^{-3} \text{ m}^3$  (2.00 L), a pressure of  $7.00 \times 10^5 \text{ Pa}$  (a gauge pressure of just under  $90.0 \text{ lb/in}^2$ ), and at a temperature of  $18.0^\circ\text{C}$  ?

**Strategy**

Identify the knowns and unknowns, and choose an equation to solve for the unknown. In this case, we solve the ideal gas law,  $PV = nRT$ , for the number of moles  $n$ .

**Solution**

- Identify the knowns.

$$P = 7.00 \times 10^5 \text{ Pa} \quad (13.40)$$

$$V = 2.00 \times 10^{-3} \text{ m}^3$$

$$T = 18.0^\circ\text{C} = 291 \text{ K}$$

$$R = 8.31 \text{ J/mol} \cdot \text{K}$$

- Rearrange the equation to solve for  $n$  and substitute known values.

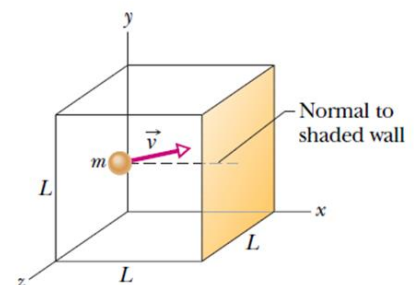
$$\begin{aligned} n &= \frac{PV}{RT} = \frac{(7.00 \times 10^5 \text{ Pa})(2.00 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol} \cdot \text{K})(291 \text{ K})} \\ &= 0.579 \text{ mol} \end{aligned} \quad (13.41)$$

## Kinetic Theory of Ideal Gas

Consider a *monatomic* ideal gas at  $V$  at  $P$  and  $T$  consists of  $N$  number of molecules moving at a speed  $v$  in a random direction. The pressure is caused by the molecule colliding on the wall.

For a typical gas molecule, of mass  $m$  and velocity  $v$ , that is about to collide with the shaded wall, as shown, if the collision with the wall is elastic, the only component of its velocity that is changed is the  $x$  component. The only change in the particle's momentum is along the  $x$  axis:

$$\Delta p_x = -2mv_x$$



**Fig. 19-4** A cubical box of edge length  $L$ , containing  $n$  moles of an ideal gas. A molecule of mass  $m$  and velocity  $\vec{v}$  is about to collide with the shaded wall of area  $L^2$ . A normal to that wall is shown.

Hence, the momentum  $\Delta p_x$  delivered to the wall by the molecule during the collision is  $+2mv_x$ .

The time  $\Delta t$  between collisions is the time the molecule takes to travel to the opposite wall and back again (a distance  $2L$ ) at speed  $v_x$ . Therefore,  $\Delta t$  is equal to  $2L/v_x$ . The force acting on the wall on average is

$$F_x = \frac{\Delta p_x}{\Delta t} = \frac{2mv_x}{2L/v_x} = \frac{mv_x^2}{L}.$$

The pressure  $P$  is given by the sum of the forces from  $N$  molecules together divided by the area.

$$P = \frac{F_x}{L^2} = \frac{m}{L^3} (v_{x1}^2 + v_{x2}^2 + \dots + v_{xN}^2) = \sum_i \frac{mv_{xi}^2}{L^3}$$

On average, each molecule contribute

$$\frac{1}{N} \sum_i \frac{mv_x^2}{L^3} \equiv \frac{m}{L^3} (v_{x_{rms}})^2$$

to the pressure. The total pressure is  $P = N \frac{mv_{x_{rms}}^2}{L^3}$

$$\text{Root-mean-Square: } v_{x_{rms}} = \sqrt{\frac{1}{N} \sum_i v_{xi}^2} = \sqrt{(v_x^2)_{avg}}$$

Instead of the x-component of the velocity, we would like to relate the pressure to the speed. Assuming the direction of the motion of each molecule is random, the average value of any direction should be the same. Therefore,  $v_{x_{rms}} = v_{y_{rms}} = v_{z_{rms}}$

$$\rightarrow (v^2)_{avg} = (v_x^2)_{avg} + (v_y^2)_{avg} + (v_z^2)_{avg} = 3(v_x^2)_{avg}$$

$$\rightarrow v_{x_{rms}}^2 = \frac{1}{3} v_{rms}^2$$

$$\text{So, } P = N \frac{mv_{rms}^2}{3L^3} = N \frac{m}{3V} v_{rms}^2$$

Combine with  $PV = nRT = NkT$

$$v_{rms}^2 = \frac{3PV}{Nm} = \frac{3kT}{m} = \frac{3RT}{M}$$

where  $M$  is the molar mass of the gas.

Total Kinetic Energy: (monatomic ideal gas)

$$K_{tot} = \frac{1}{2} m \sum_i v_i^2 = \frac{1}{2} mN (v^2)_{avg} = \frac{1}{2} mN v_{rms}^2 = \frac{3}{2} NkT = \frac{3}{2} nRT$$

For ideal gas, the internal energy equals to the total kinetic energy of the molecules.



Average kinetic energy per molecule =  $K_{\text{tot}}/N = \frac{3}{2}kT$

### Example 13.10 Calculating Kinetic Energy and Speed of a Gas Molecule

(a) What is the average kinetic energy of a gas molecule at 20.0°C (room temperature)? (b) Find the rms speed of a nitrogen molecule ( $N_2$ ) at this temperature.

#### Strategy for (a)

The known in the equation for the average kinetic energy is the temperature.

$$\overline{KE} = \frac{1}{2}mv^2 = \frac{3}{2}kT \quad (13.57)$$

Before substituting values into this equation, we must convert the given temperature to kelvins. This conversion gives  $T = (20.0 + 273) \text{ K} = 293 \text{ K}$ .

#### Solution for (a)

The temperature alone is sufficient to find the average translational kinetic energy. Substituting the temperature into the translational kinetic energy equation gives

$$\overline{KE} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = 6.07 \times 10^{-21} \text{ J.} \quad (13.58)$$

#### Strategy for (b)

Finding the rms speed of a nitrogen molecule involves a straightforward calculation using the equation

$$\sqrt{\frac{3}{2}kT} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}, \quad (13.59)$$

but we must first find the mass of a nitrogen molecule. Using the molecular mass of nitrogen  $N_2$  from the periodic table,

$$m = \frac{2(14.0067) \times 10^{-3} \text{ kg/mol}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 4.65 \times 10^{-26} \text{ kg.} \quad (13.60)$$

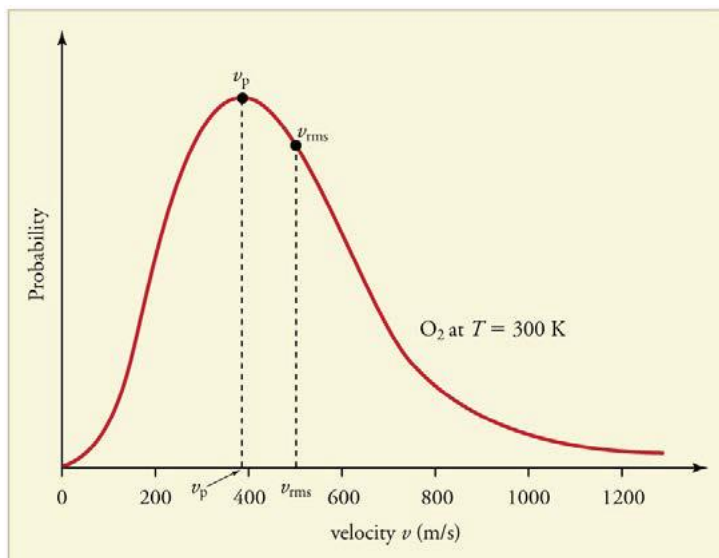
#### Solution for (b)

Substituting this mass and the value for  $k$  into the equation for  $v_{\text{rms}}$  yields

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{4.65 \times 10^{-26} \text{ kg}}} = 511 \text{ m/s.} \quad (13.61)$$

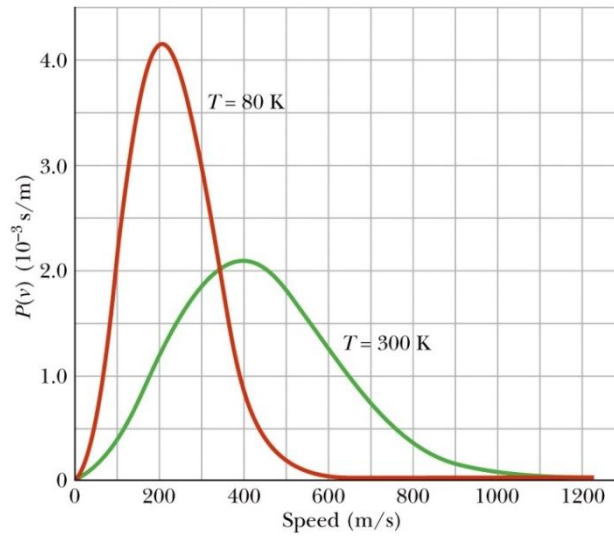
## Distribution of molecular speed

The motion of molecules in a gas is random in magnitude and direction for individual molecules, but a gas of many molecules has a predictable distribution of molecular speeds. This distribution is called the **Maxwell-Boltzmann distribution**, after its originators, who calculated it based on kinetic theory, and has since been confirmed experimentally. The distribution has a long tail, because a few molecules may go several times the  $v_{\text{rms}}$  speed. The most probable speed  $v_p$  is less than the rms speed  $v_{\text{rms}}$ .



The distribution of thermal speeds depends strongly on temperature. As temperature increases, the speeds are shifted to higher values and the distribution is broadened; therefore, more chance to find

a molecule moving at high speed.



### Example 13.11 Calculating Temperature: Escape Velocity of Helium Atoms

In order to escape Earth's gravity, an object near the top of the atmosphere (at an altitude of 100 km) must travel away from Earth at 11.1 km/s. This speed is called the *escape velocity*. At what temperature would helium atoms have an rms speed equal to the escape velocity?

#### Strategy

Identify the knowns and unknowns and determine which equations to use to solve the problem.

#### Solution

1. Identify the knowns:  $v$  is the escape velocity, 11.1 km/s.
2. Identify the unknowns: We need to solve for temperature,  $T$ . We also need to solve for the mass  $m$  of the helium atom.
3. Determine which equations are needed.

- To solve for mass  $m$  of the helium atom, we can use information from the periodic table:

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}} \quad (13.62)$$

- To solve for temperature  $T$ , we can rearrange either

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT \quad (13.63)$$

or

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}} \quad (13.64)$$

to yield

$$T = \frac{mv_{\text{rms}}^2}{3k}, \quad (13.65)$$

where  $k$  is the Boltzmann constant and  $m$  is the mass of a helium atom.

4. Plug the known values into the equations and solve for the unknowns.

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}} = \frac{4.0026 \times 10^{-3} \text{ kg/mol}}{6.02 \times 10^{23} \text{ mol}} = 6.65 \times 10^{-27} \text{ kg} \quad (13.66)$$

$$T = \frac{(6.65 \times 10^{-27} \text{ kg})(11.1 \times 10^3 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 1.98 \times 10^4 \text{ K} \quad (13.67)$$