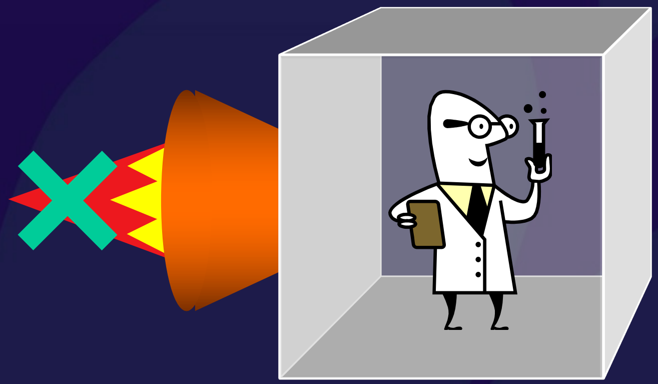
Introduction to Relativity - 1

Galilean invariance

We have introduced the concept of inertial reference frame



inertial frame

- Newton's Law is valid in all reference frames that are not accelerating, or are not themselves act upon by forces (Galilean invariance).

The important of Galilean invariance can be seen from the following example:

This is a very big boat, and is very stable so that you do not feel any floating up-and-down motion of the boat.

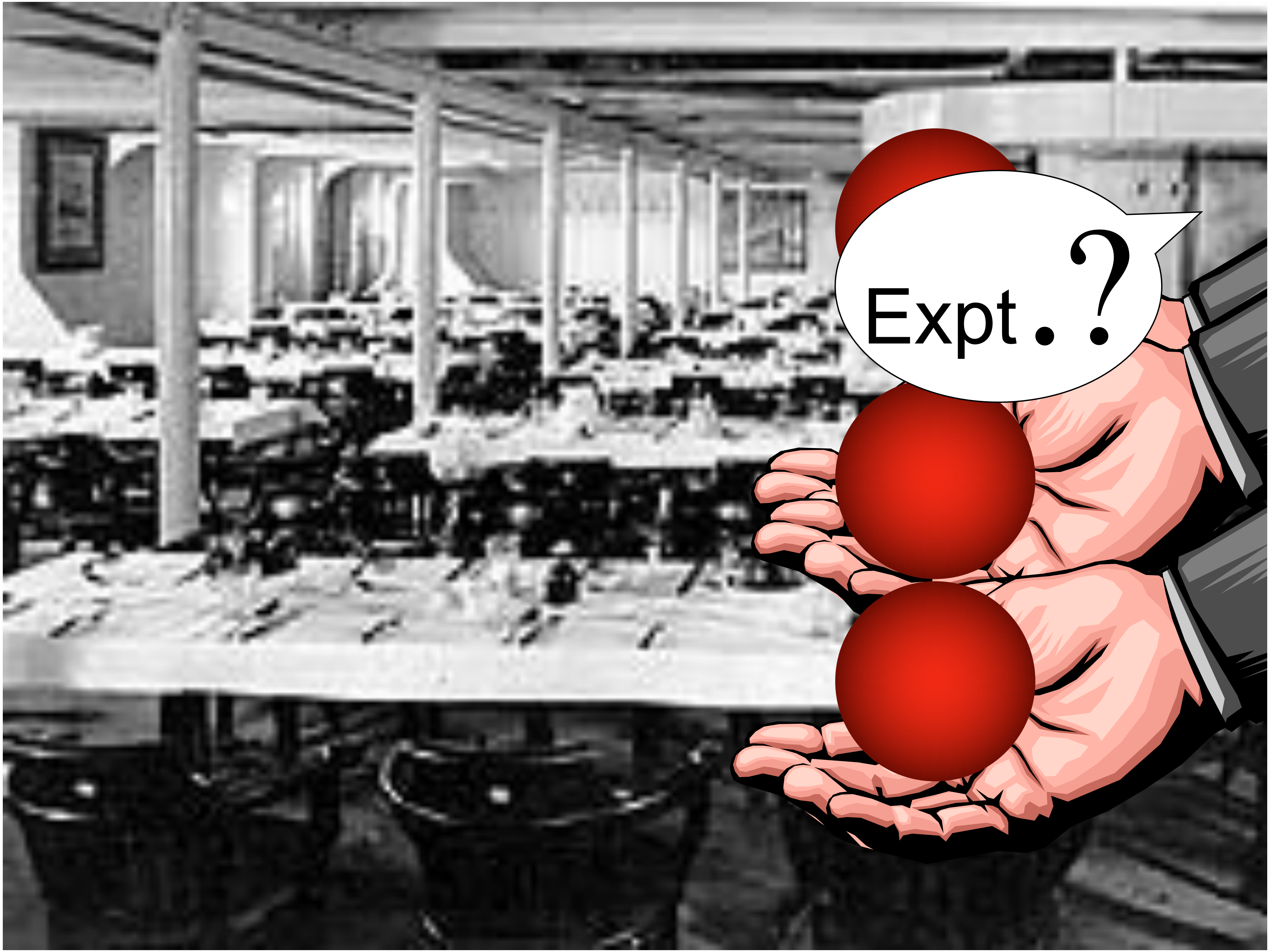
Suppose you are living at the interior of the boat and have no window to look outside.

One day, you wake up from a nap, and out of curiosity you want to find out whether the boat is…

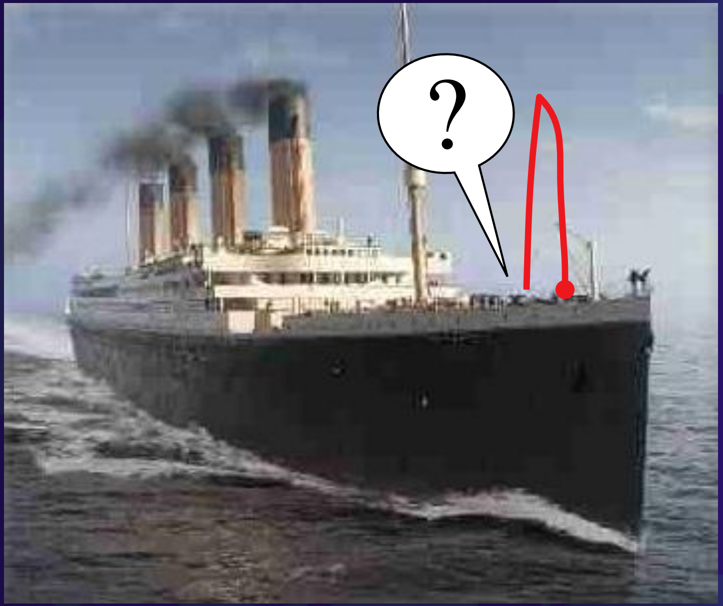
actually parking somewhere,

or is traveling with a uniform speed.

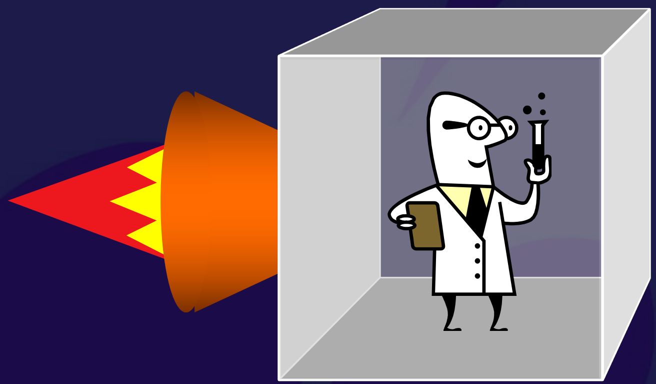
Can you find that out without asking someone, and without going outside?


Expt.?

What Galilean invariance said is that you cannot determine whether you are moving (**with uniform velocity**) or not if you perform experiments involving Newton's Law inside the boat, like looking at the trajectory of a little ball that you throw up, or looking at the motion of anything you see.

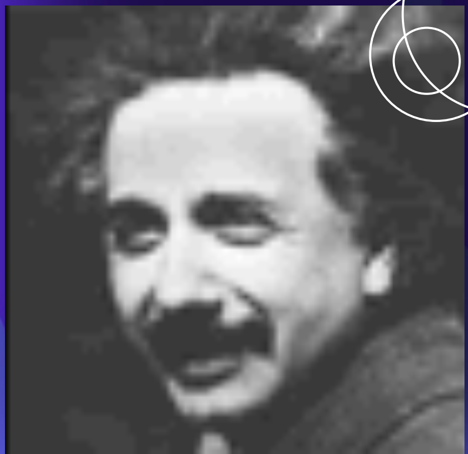


Natural Question: How about if the boat is accelerating? Do you think you can detect that inside the room?



Yes, because inertial (pseudo) forces will be observed

Is there any fundamental reason why Newton's Law cannot detect uniform motions? Is it possible that it is a fundamental nature of our universe that no matter what tools (physical phenomena) you use, you cannot tell whether something is moving with uniform velocity or at rest (with respect to our universe)?



This amazing fact is what inspires Einstein to ask the following question

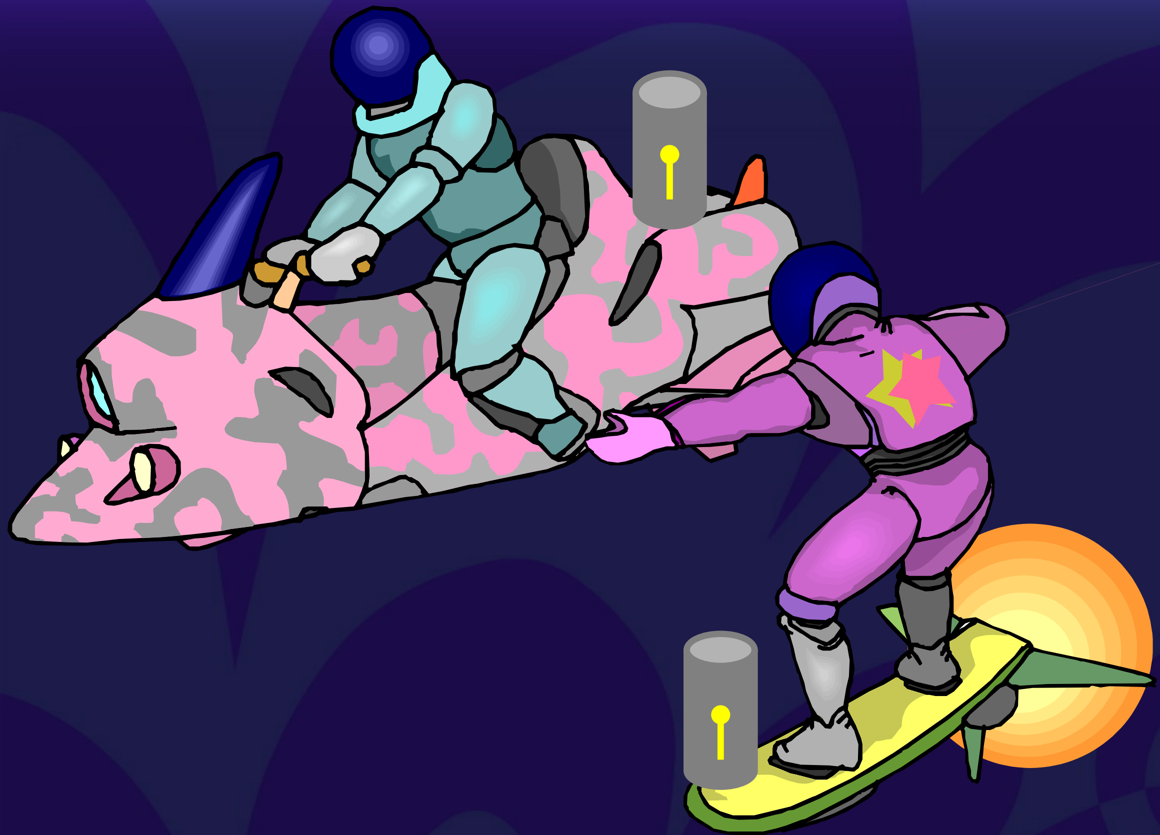
He investigate the consequence if we cannot distinguish uniform motion from rest using *any* E&M (light) experiments.

The answer he obtained was revolutionary:

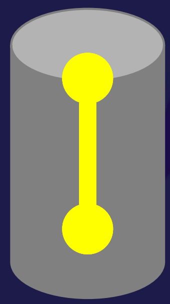
If we require that the physical laws of nature have the absolute property that they

cannot distinguish uniform motions from rest,

then distance and time must become relative.

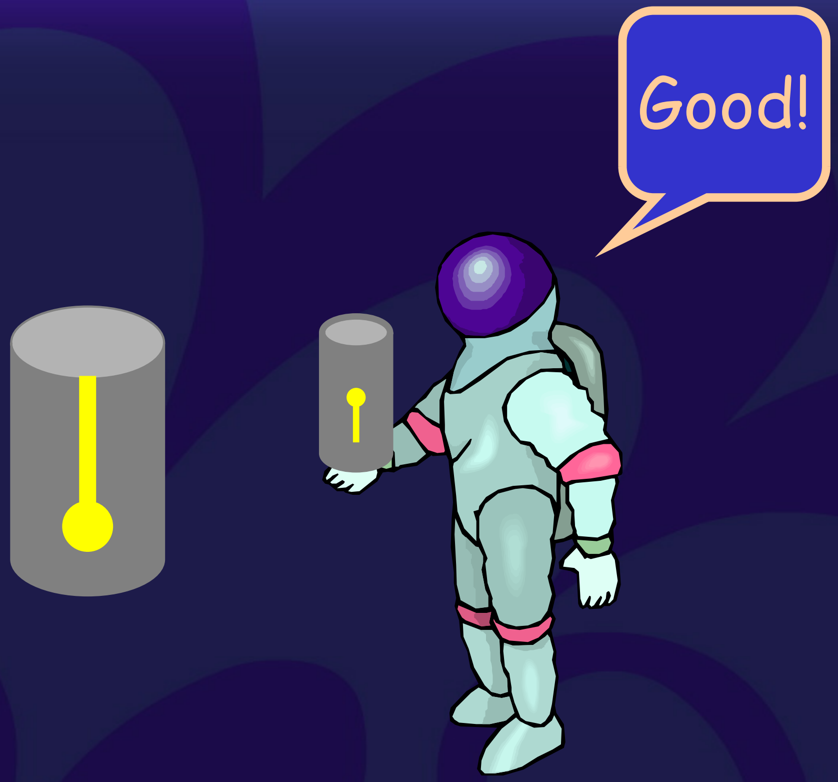


The length of time between two events and the distance between two events that take place depends on the status of the observer.

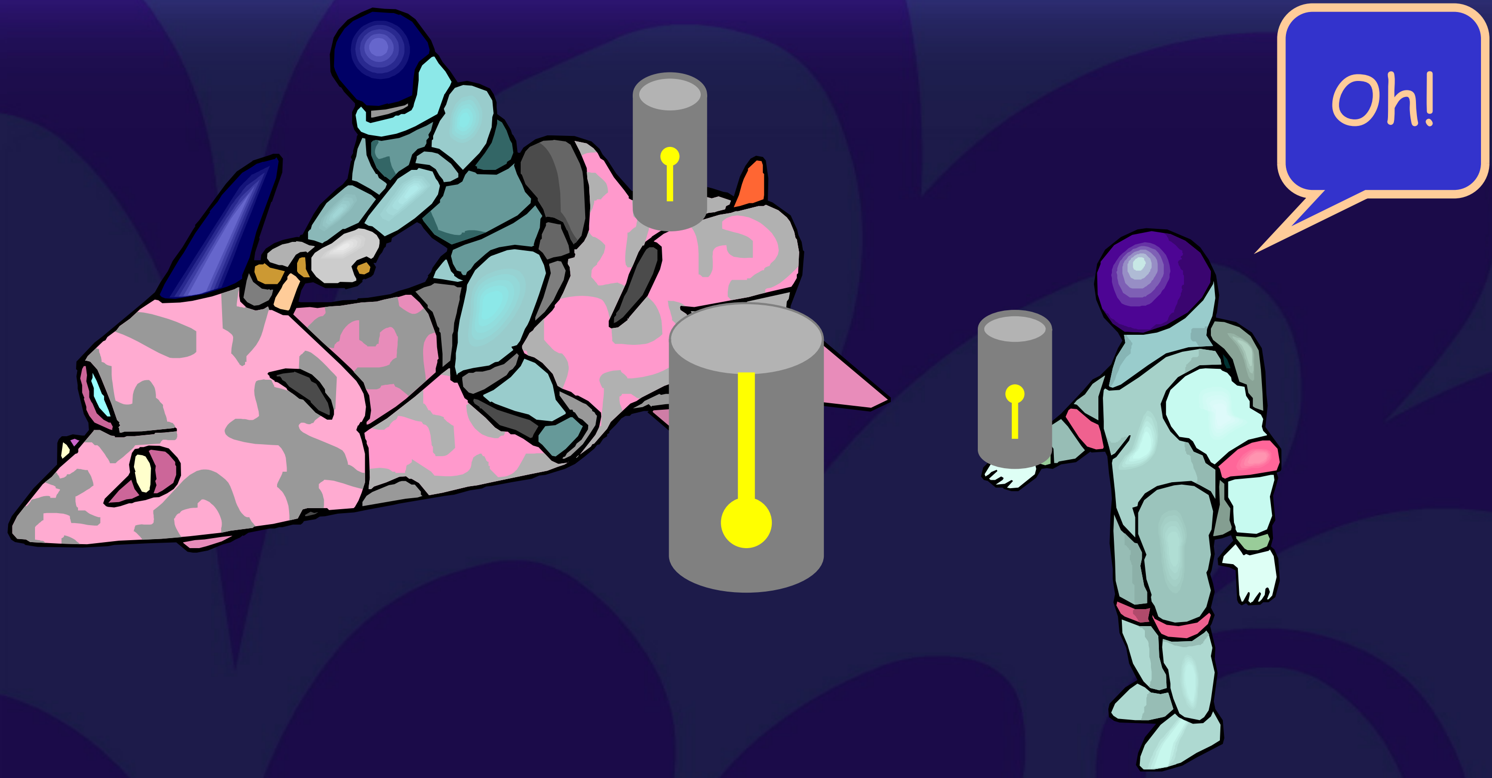


This is the essence of the

theory of special relativity

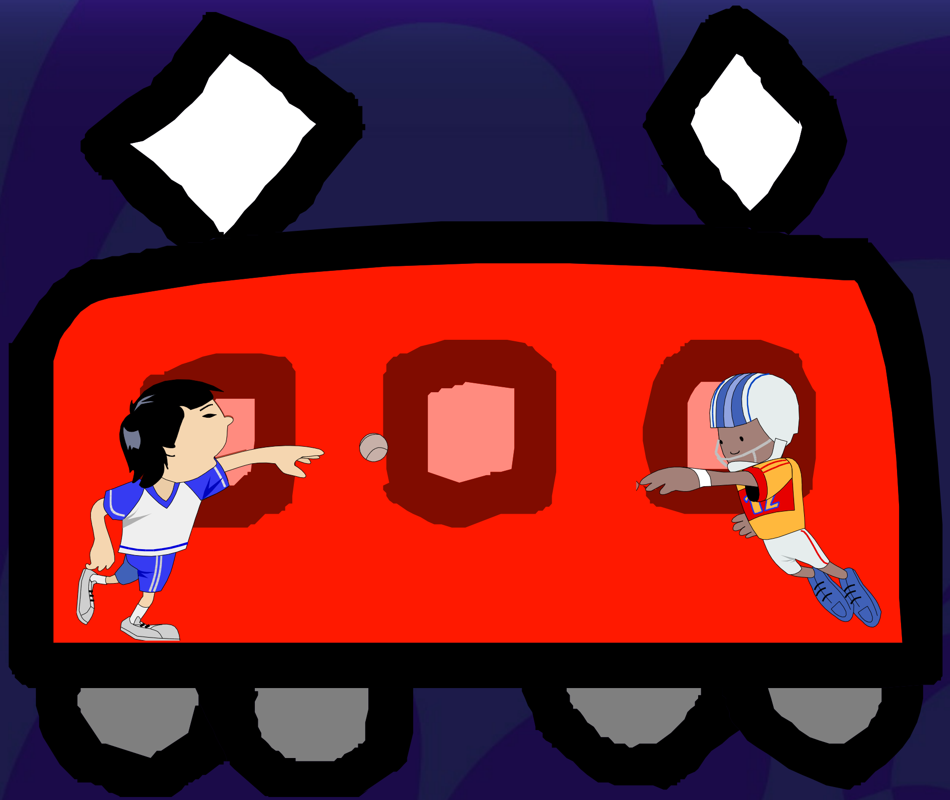


The relativeness of space and time has only very small effect when the relative speed between two objects is much less than speed of light.



However, the effect becomes drastic when the (relative) speed of motion is close to speed of light.

Electromagnetism and Special Relativity

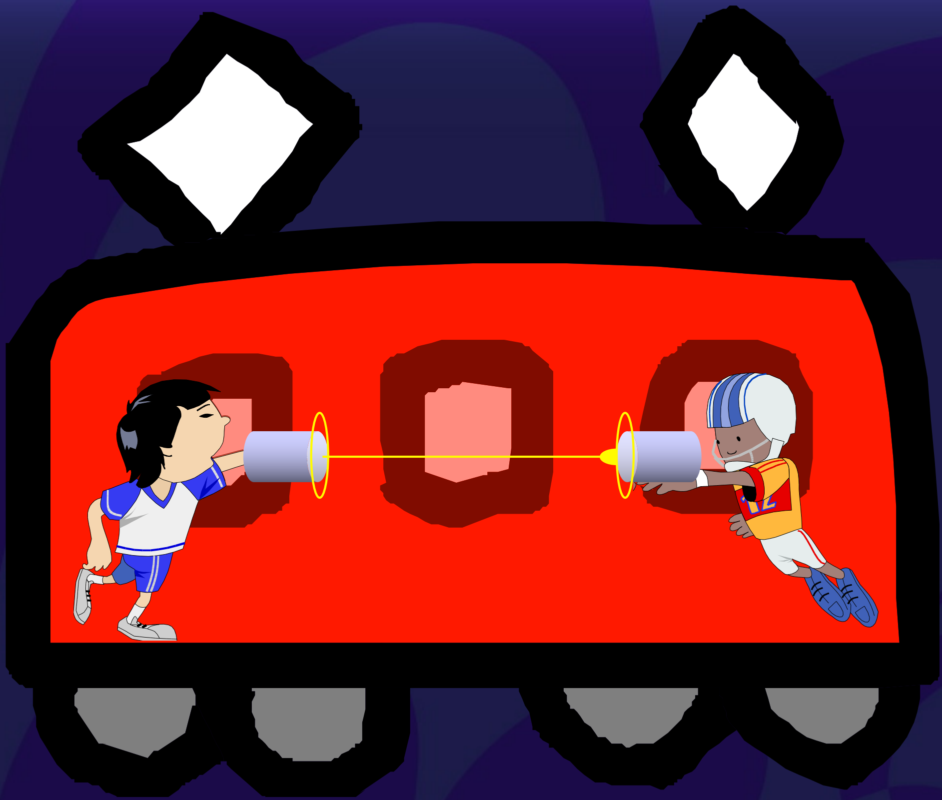


Throwing a ball in a train,

can you tell whether the train is moving with uniform velocity or at rest?

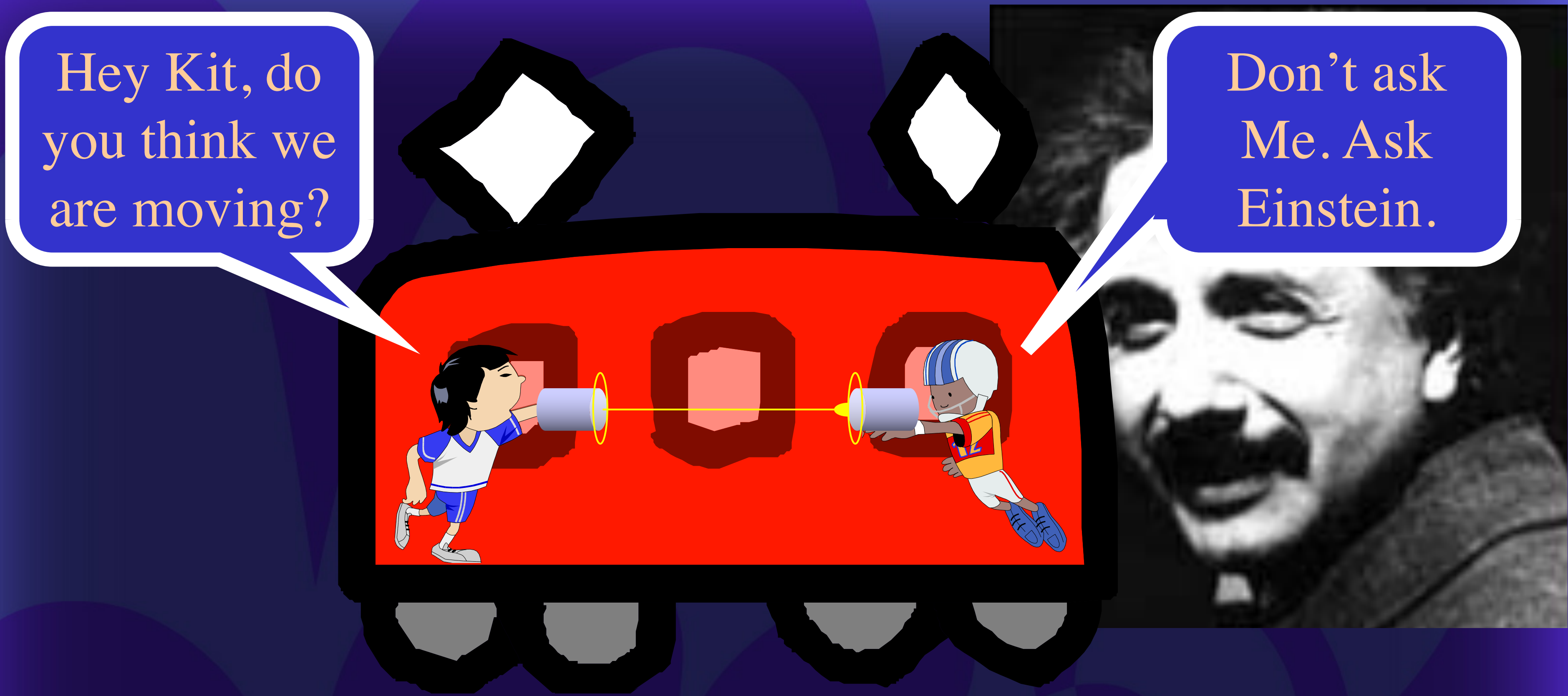
No you can't.

(by Galileo relativity)



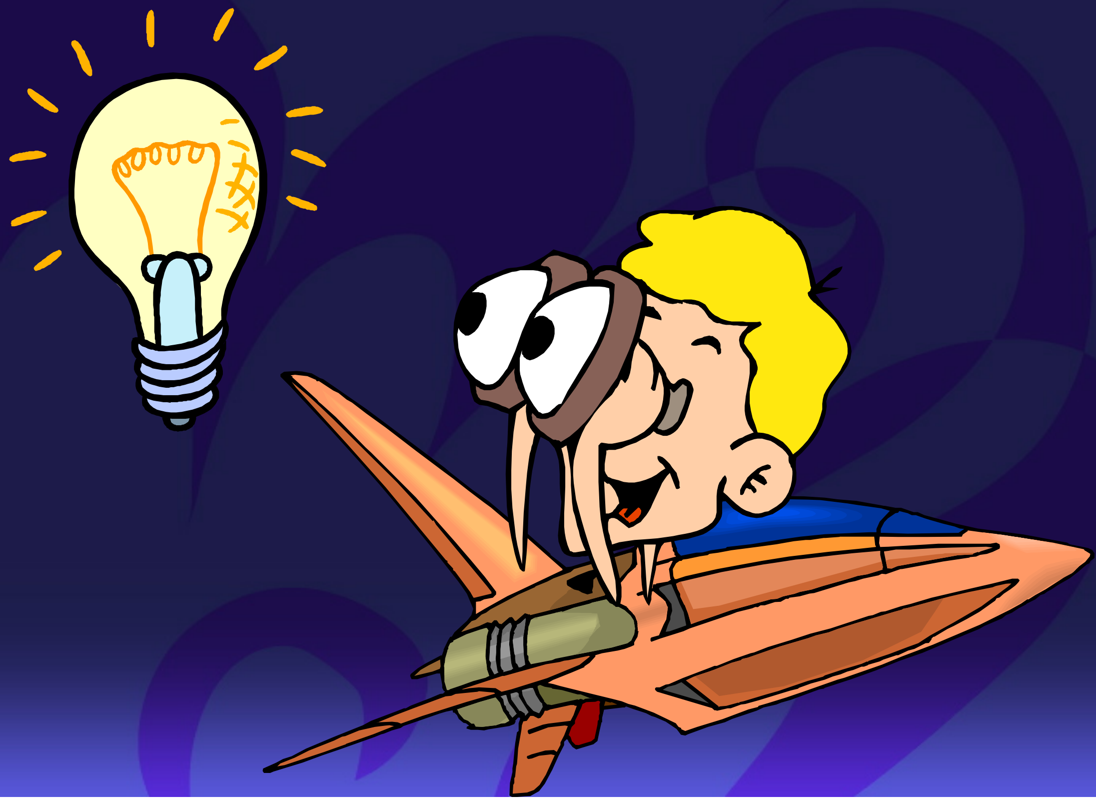
How about sending a light pulse?

Einstein's answer: " still you can't tell".



Einstein realized that if Maxwell equations appear the same in all inertial reference frames, than uniform motion could not be detected by experiments involving electricity and magnetism.

In particular, Einstein noticed that if Maxwell equation is the same in all inertial reference frames, the speed of light will be the same in all inertial reference frames, independent of the motion of the object sending and receiving the light signal!

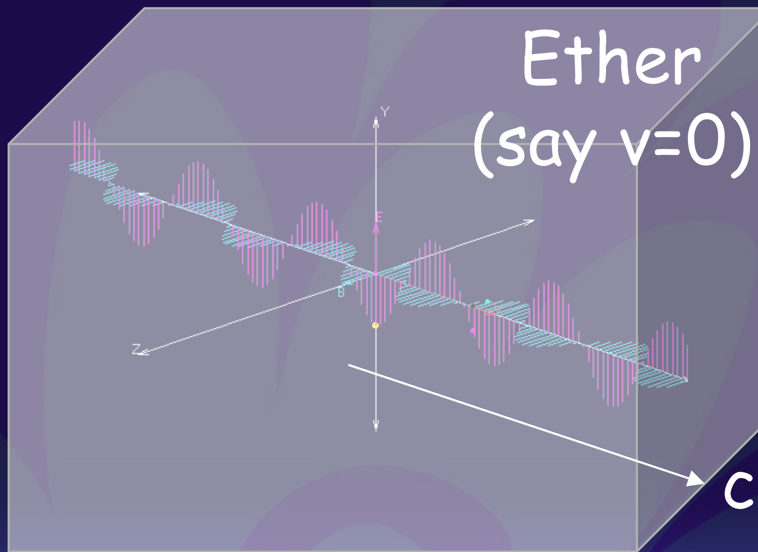


For example, the speed of light (relative to you) is the same independent of whether you are moving along the same direction or opposite direction of the light beam.

Before Einstein, some people believed that EM waves needs a medium, called Ether, to propagate.

Because EM waves can reach all places, it is believed that it filled up our universe.

The speed of light, *c, is the velocity the EM wave travels relative to this fixed media .*



To someone who is moving in Ether, the speed of light will be different.

After Einstein, most people believe that there is no Ether

EM waves can propagate in vacuum, and the speed of light is the **same** for all observers.

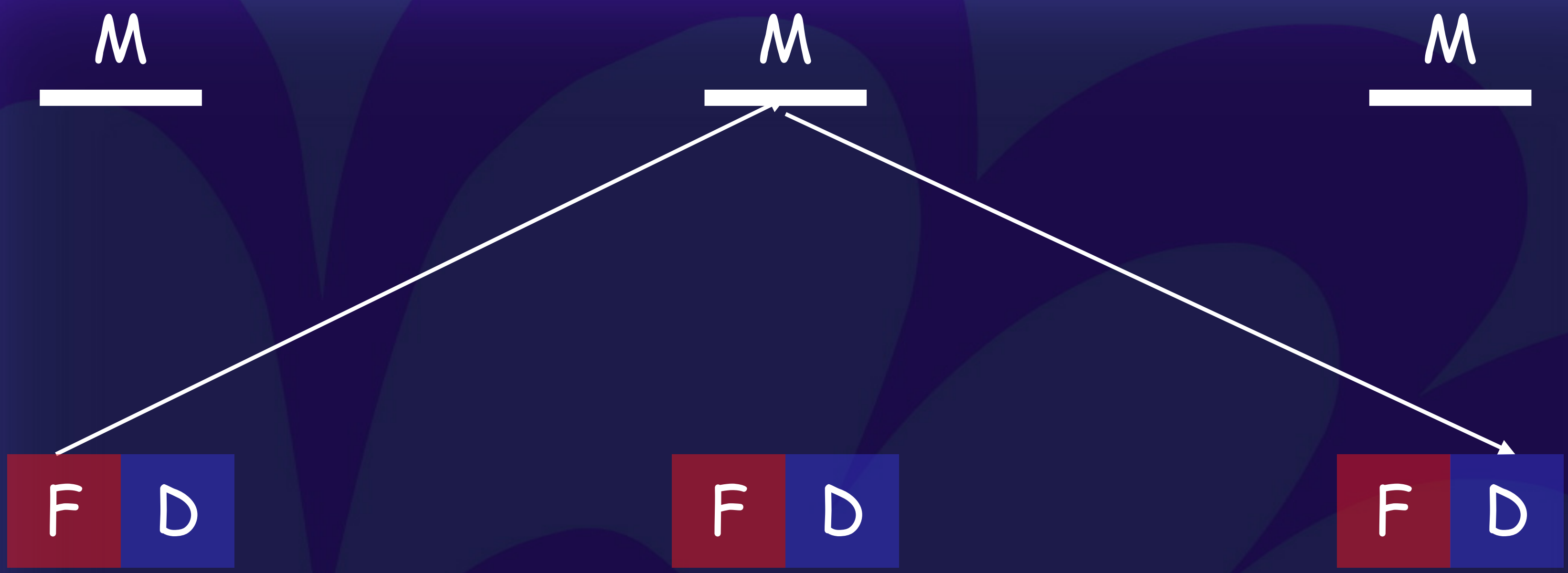
Time Dilation when speed of light is the same at all inertial reference frames

Let us consider the following simple experiment.

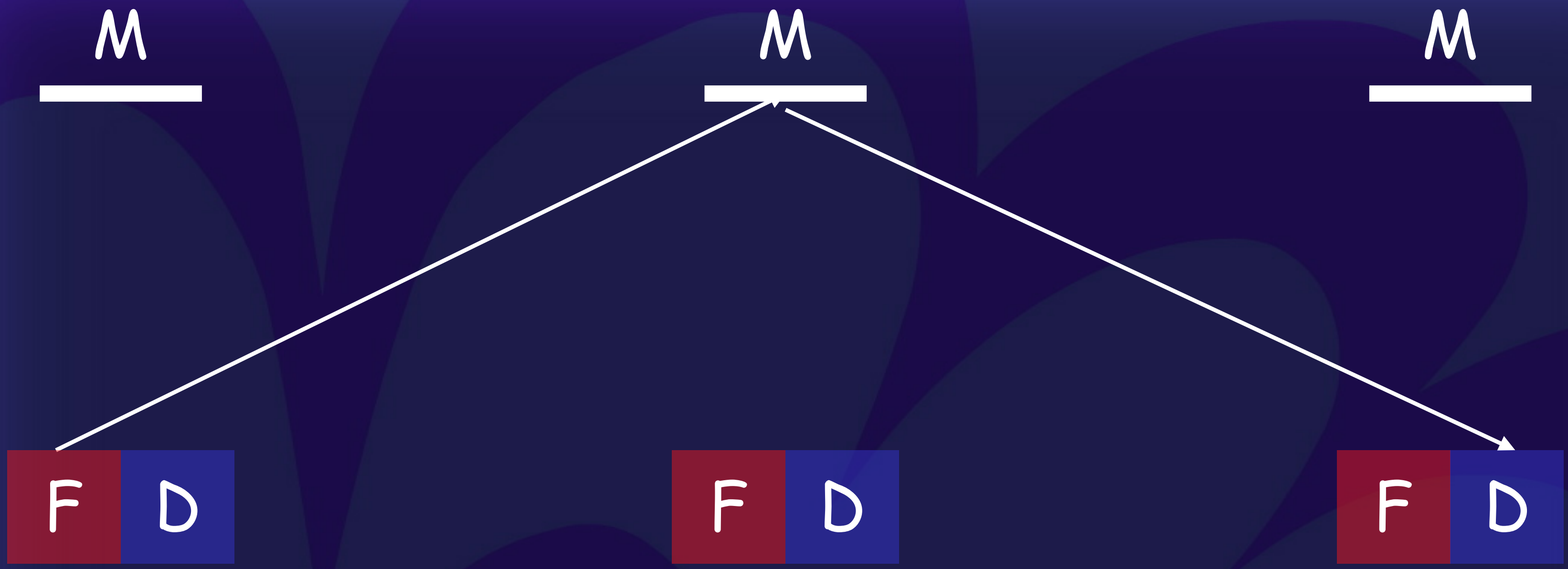
A flashing light bulb F is attached to a detector D and separated by a distance L_0 from a mirror M. A flash of light from the bulb is reflected back by the mirror, and is detected by D. The time interval between emission of detection is

$$t_0 = \frac{2L_0}{c}$$

according to an observer at rest with the device.

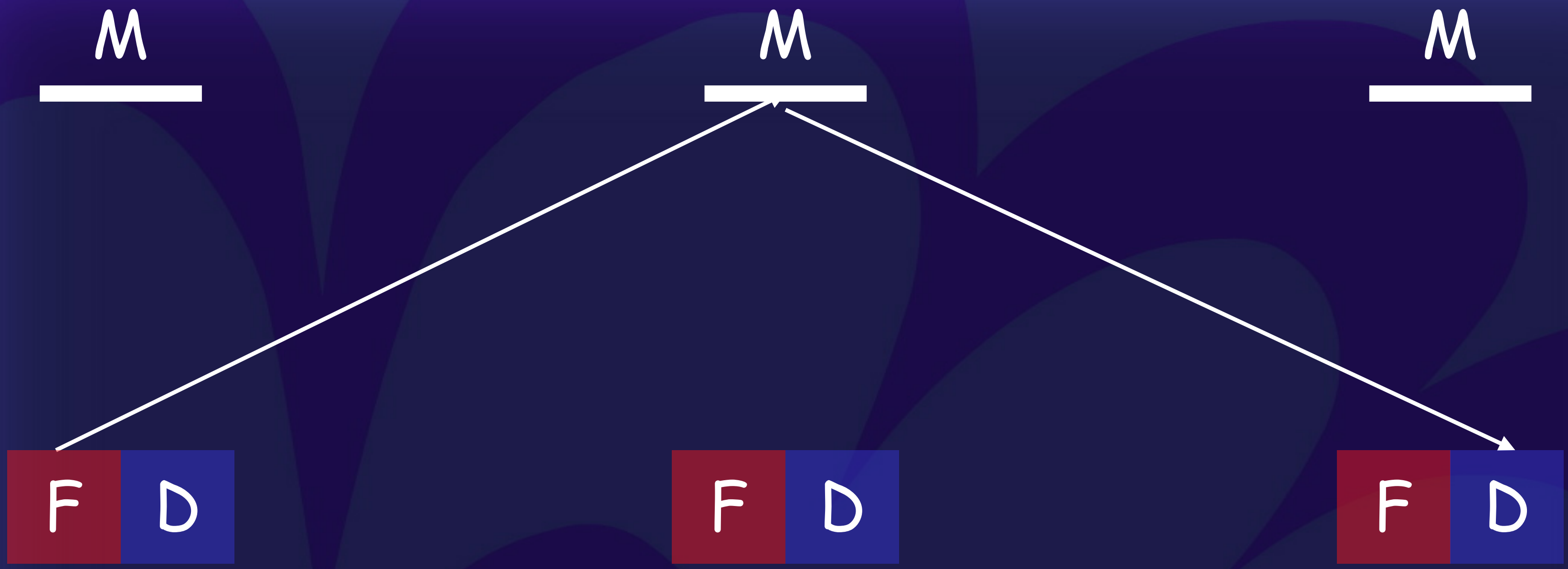


Now let us examine the *same experiment* again from another observer who is travelling from right to left with velocity u. According to this observer, the whole device is moving from left to right with velocity u.



Let us denote the time between the emission and detection of light be Δt for the second observer. The whole device has traveled a distance uΔt and the total distance light has traveled is

$$\sqrt{4L_0^2 + u^2(\Delta t)^2}$$



If the speed of light is still c, then

$$\Delta t = \frac{\sqrt{4L_0^2 + u^2(\Delta t)^2}}{c} = \sqrt{\left(\frac{2L_0}{c}\right)^2 + \left(\frac{u}{c}\right)^2 (\Delta t)^2} = \sqrt{(\Delta t_0)^2 + \left(\frac{u}{c}\right)^2 (\Delta t)^2}$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{u}{c}\right)^2}}$$

Time dilation and proper time

- Let Δt_0 be the **proper time** between two events (i.e. the two events occur at the same position).
- An observer moving with constant speed u will measure the time interval to be Δt, where

Time dilation:

Proper time between two events (measured in rest frame)

$$\Delta t = \gamma \Delta t_0$$

Lorentz factor relating the two frames

Time interval between same events measured in second frame of reference

where the Lorentz factor γ is defined as:

Lorentz factor

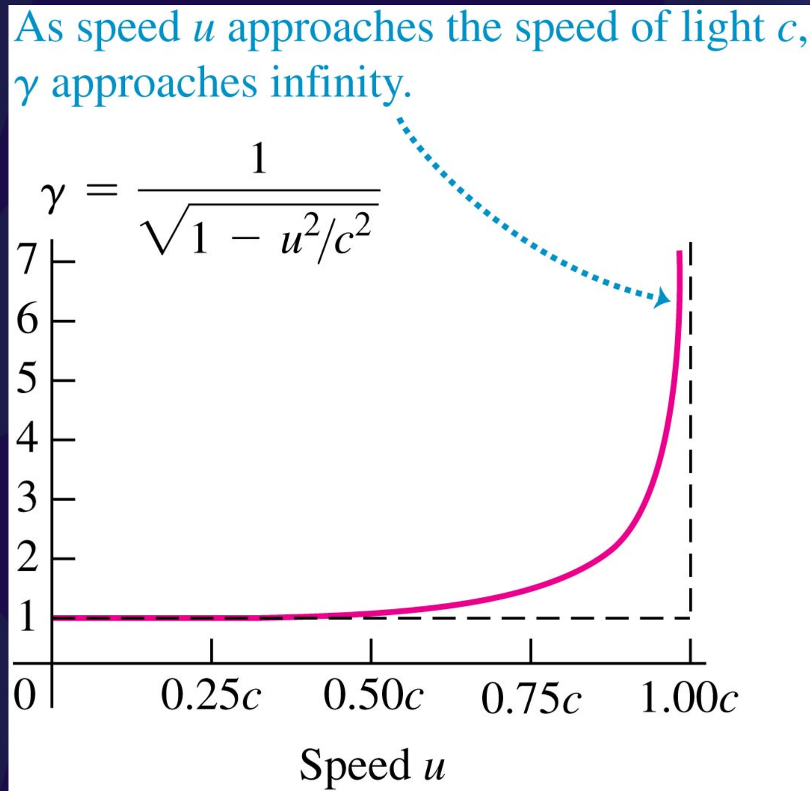
$$\gamma = \frac{1}{\sqrt{1 - u^2/c^2}}$$

Speed of light in vacuum

Speed of one frame of reference relative to another

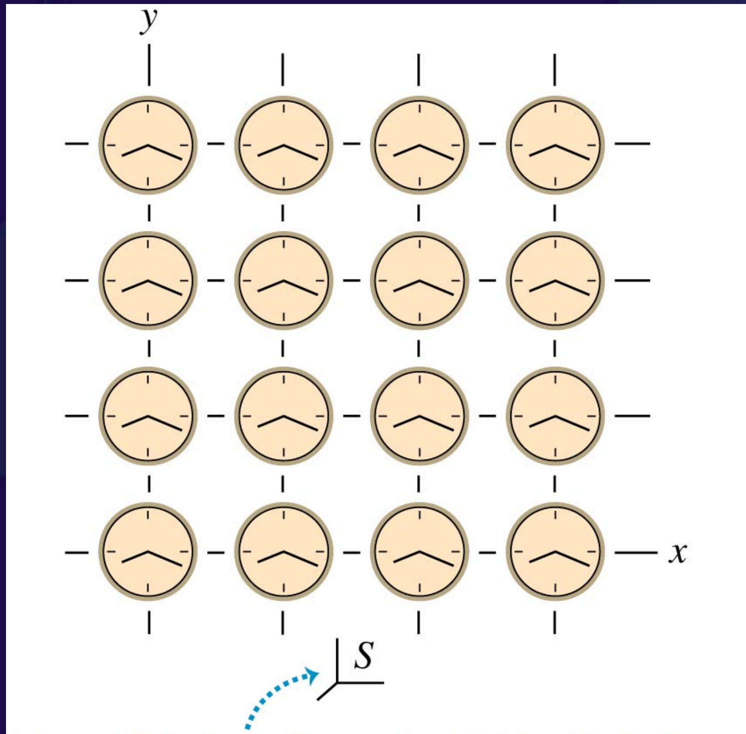
The Lorentz factor

- When u is very small compared to c, γ is very nearly equal to 1.
- If the relative speed u is great enough that γ is appreciably greater than 1, the speed is said to be relativistic.



Proper time

- *Proper time* is the time interval between two events that occur at the *same point (at rest)*.
- A frame of reference can be pictured as a coordinate system with a grid of synchronized clocks, as in the figure at the right.



The grid is three dimensional; identical planes of clocks lie in front of and behind the page, connected by grid lines perpendicular to the page.

Example 37.1 Time dilation at 0.990*c*

High-energy subatomic particles coming from space interact with atoms in the earth's upper atmosphere, in some cases producing unstable particles called *muons*. A muon decays into other particles with a mean lifetime of $2.20\ \mu s = 2.20 \times 10^{-6}$ s as measured in a reference frame in which it is at rest. If a muon is moving at $0.990c$ relative to the earth, what will an observer on earth measure its mean lifetime to be?

SOLUTION

IDENTIFY and SET UP: The muon's lifetime is the time interval between two events: the production of the muon and its subsequent decay. Our target variable is the lifetime in your frame of reference on earth, which we call frame S. We are given the lifetime in a frame S' in which the muon is at rest; this is its *proper* lifetime, $\Delta t_0 = 2.20\ \mu s$. The relative speed of these two frames is $u = 0.990c$. We use Eq. (37.6) to relate the lifetimes in the two frames.

EXECUTE: The muon moves relative to the earth between the two events, so the two events occur at different positions as measured in S and the time interval in that frame is Δt (the target variable). From Eq. (37.6),

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - u^2/c^2}} = \frac{2.20\ \mu s}{\sqrt{1 - (0.990)^2}} = 15.6\ \mu s$$

EVALUATE: Our result predicts that the mean lifetime of the muon in the earth frame (Δt) is about seven times longer than in the muon's frame (Δt_0). This prediction has been verified experimentally; indeed, this was the first experimental confirmation of the time dilation formula, Eq. (37.6).

Example 37.2 Time dilation at airliner speeds

An airplane flies from San Francisco to New York (about 4800 km, or 4.80×10^6 m) at a steady speed of 300 m/s (about 670 mi/h). How much time does the trip take, as measured by an observer on the ground? By an observer in the plane?

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SOLUTION

IDENTIFY and SET UP: Here we're interested in the time interval between the airplane departing from San Francisco and landing in New York. The target variables are the time intervals as measured in the frame of reference of the ground S and in the frame of reference of the airplane S'.

EXECUTE: As measured in S the two events occur at different positions (San Francisco and New York), so the time interval measured by ground observers corresponds to Δt in Eq. (37.6). To find it, we simply divide the distance by the speed $u = 300$ m/s:

$$\Delta t = \frac{4.80 \times 10^6\ \text{m}}{300\ \text{m/s}} = 1.60 \times 10^4\ \text{s} \quad (\text{about } 4\tfrac{1}{2}\ \text{hours})$$

In the airplane's frame S', San Francisco and New York passing under the plane occur at the same point (the position of the plane). Hence the time interval in the airplane is a proper time, corresponding to Δt_0 in Eq. (37.6). We have

$$\frac{u^2}{c^2} = \frac{(300\ \text{m/s})^2}{(3.00 \times 10^8\ \text{m/s})^2} = 1.00 \times 10^{-12}$$

From Eq. (37.6),

$$\Delta t_0 = (1.60 \times 10^4\ \text{s})\sqrt{1 - 1.00 \times 10^{-12}}$$

The square root can't be evaluated with adequate precision with an ordinary calculator. But we can approximate it using the binomial theorem (see Appendix B):

$$(1 - 1.00 \times 10^{-12})^{1/2} = 1 - \left(\tfrac{1}{2}\right)(1.00 \times 10^{-12}) + \cdots$$

The remaining terms are of the order of 10^{-24} or smaller and can be discarded. The approximate result for Δt_0 is

$$\Delta t_0 = (1.60 \times 10^4\ \text{s})(1 - 0.50 \times 10^{-12})$$

The proper time Δt_0, measured in the airplane, is very slightly less (by less than one part in 10^{12}) than the time measured on the ground.

EVALUATE: We don't notice such effects in everyday life. But present-day atomic clocks (see Section 1.3) can attain a precision of about one part in 10^{13}. A cesium clock traveling a long distance in an airliner has been used to measure this effect and thereby verify Eq. (37.6) even at speeds much less than c.

Example 37.3 Just when is it proper?

Mavis boards a spaceship and then zips past Stanley on earth at a relative speed of $0.600c$. At the instant she passes him, they both start timers. (a) A short time later Stanley measures that Mavis has traveled 9.00×10^7 m beyond him and is passing a space station. What does Stanley's timer read as she passes the space station? What does Mavis's timer read? (b) Stanley starts to blink just as Mavis flies past him, and Mavis measures that the blink takes 0.400 s from beginning to end. According to Stanley, what is the duration of his blink?

SOLUTION

IDENTIFY and SET UP: This problem involves time dilation for two *different* sets of events measured in Stanley's frame of reference (which we call S) and in Mavis's frame of reference (which we call S'). The two events of interest in part (a) are when Mavis passes Stanley and when Mavis passes the space station; the target variables are the time intervals between these two events as measured in S and in S'. The two events in part (b) are the start and finish of Stanley's blink; the target variable is the time interval between these two events as measured in S.

EXECUTE: (a) The two events, Mavis passing the earth and Mavis passing the space station, occur at different positions in Stanley's frame but at the same position in Mavis's frame. Hence Stanley measures time interval Δt, while Mavis measures the *proper* time Δt_0. As measured by Stanley, Mavis moves at $0.600c = 0.600(3.00 \times 10^8 \text{ m/s}) = 1.80 \times 10^8 \text{ m/s}$ and travels 9.00×10^7 m in time $\Delta t = (9.00 \times 10^7 \text{ m})/(1.80 \times 10^8 \text{ m/s}) = 0.500$ s. From Eq. (37.6), Mavis's timer reads an elapsed time of

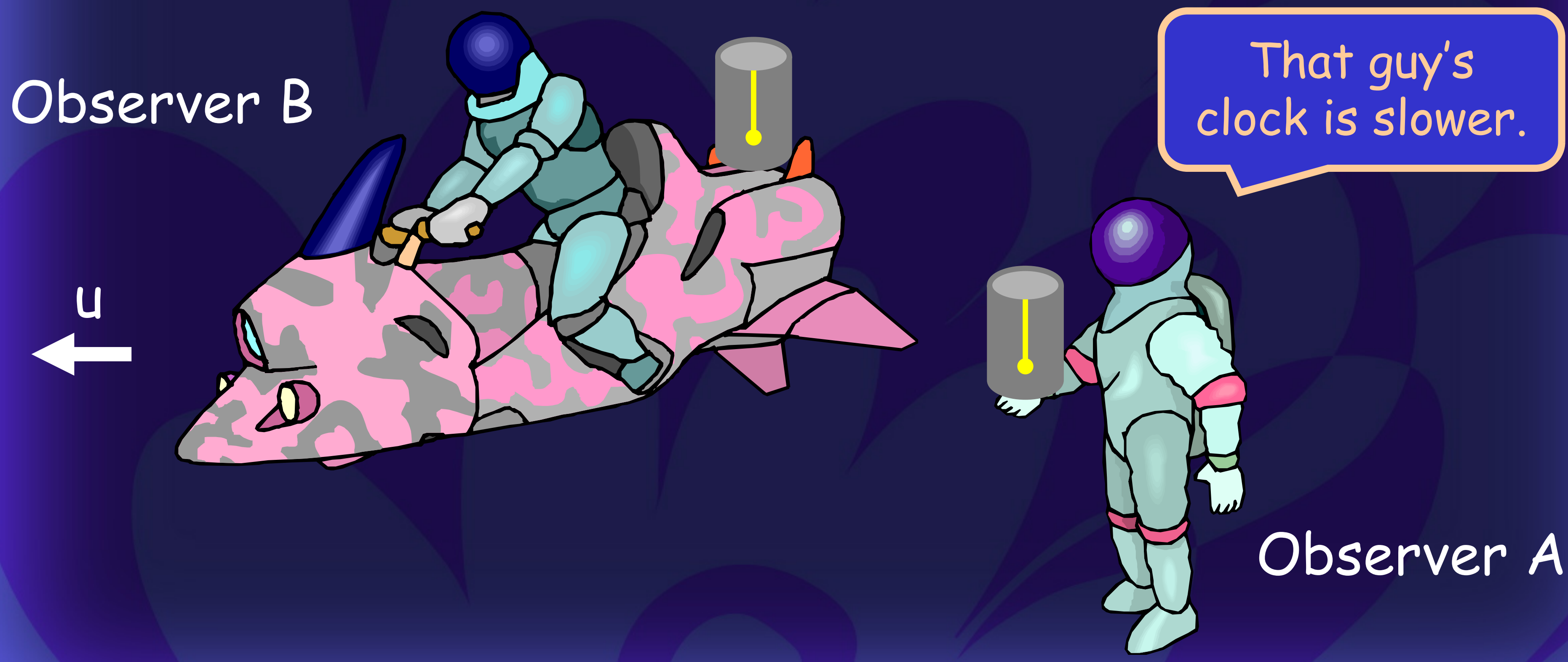
$$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = 0.500 \text{ s} \sqrt{1 - (0.600)^2} = 0.400 \text{ s}$$

(b) It is tempting to answer that Stanley's blink lasts 0.500 s in his frame. But this is wrong, because we are now considering a *different* pair of events than in part (a). The start and finish of Stanley's blink occur at the same point in his frame S but at different positions in Mavis's frame S', so the time interval of 0.400 s that she measures between these events is equal to Δt. The duration of the blink measured on Stanley's timer is the proper time Δt_0:

$$\Delta t_0 = \Delta t \sqrt{1 - u^2/c^2} = 0.400 \text{ s} \sqrt{1 - (0.600)^2} = 0.320 \text{ s}$$

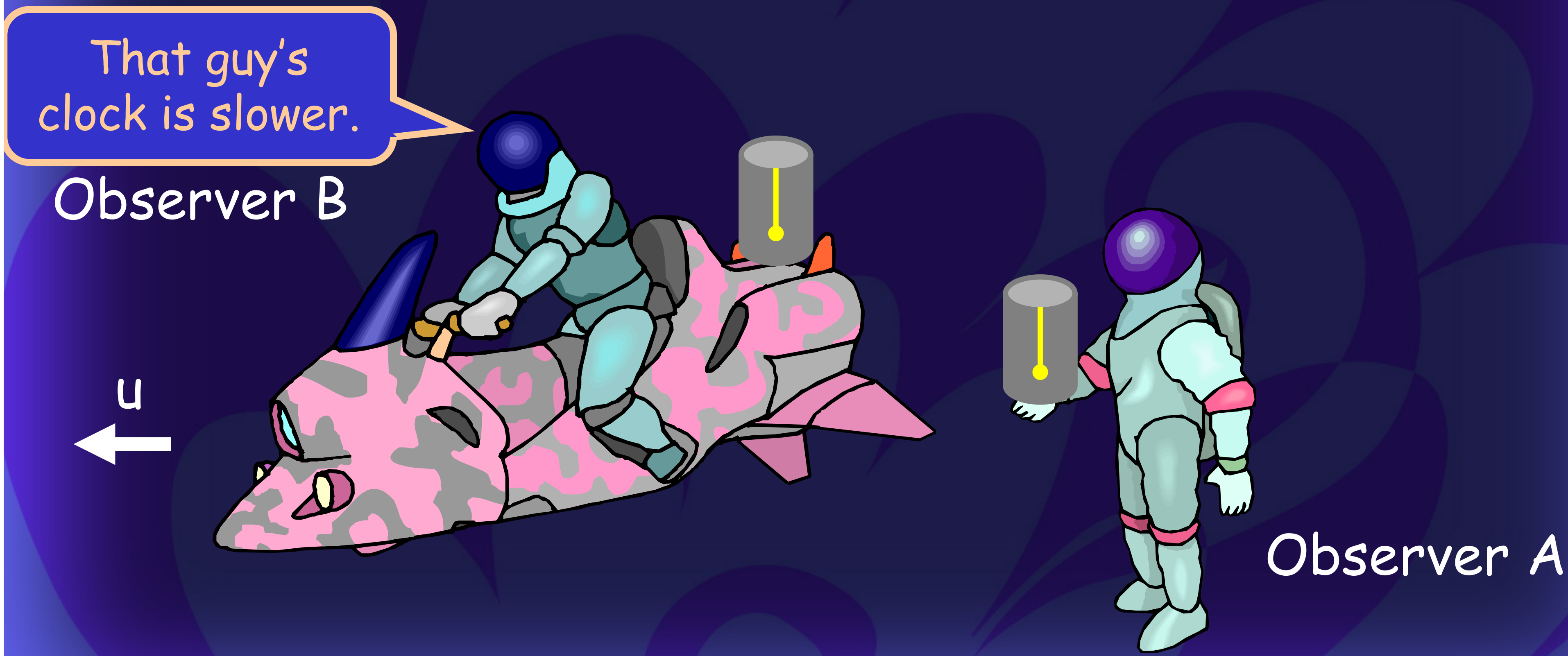
EVALUATE: This example illustrates the relativity of simultaneity. In Mavis's frame she passes the space station at the same instant that Stanley finishes his blink, 0.400 s after she passed Stanley. Hence these two events are simultaneous to Mavis in frame S'. But these two events are *not* simultaneous to Stanley in his frame S: According to his timer, he finishes his blink after 0.320 s and Mavis passes the space station after 0.500 s.

An event that takes time Δt_0 if it is at rest with respect to the observer will take longer time if it is travelling with respect to the observer, i.e. *time seems to run slower (dilated) for an object which is travelling with respect to the observer!*

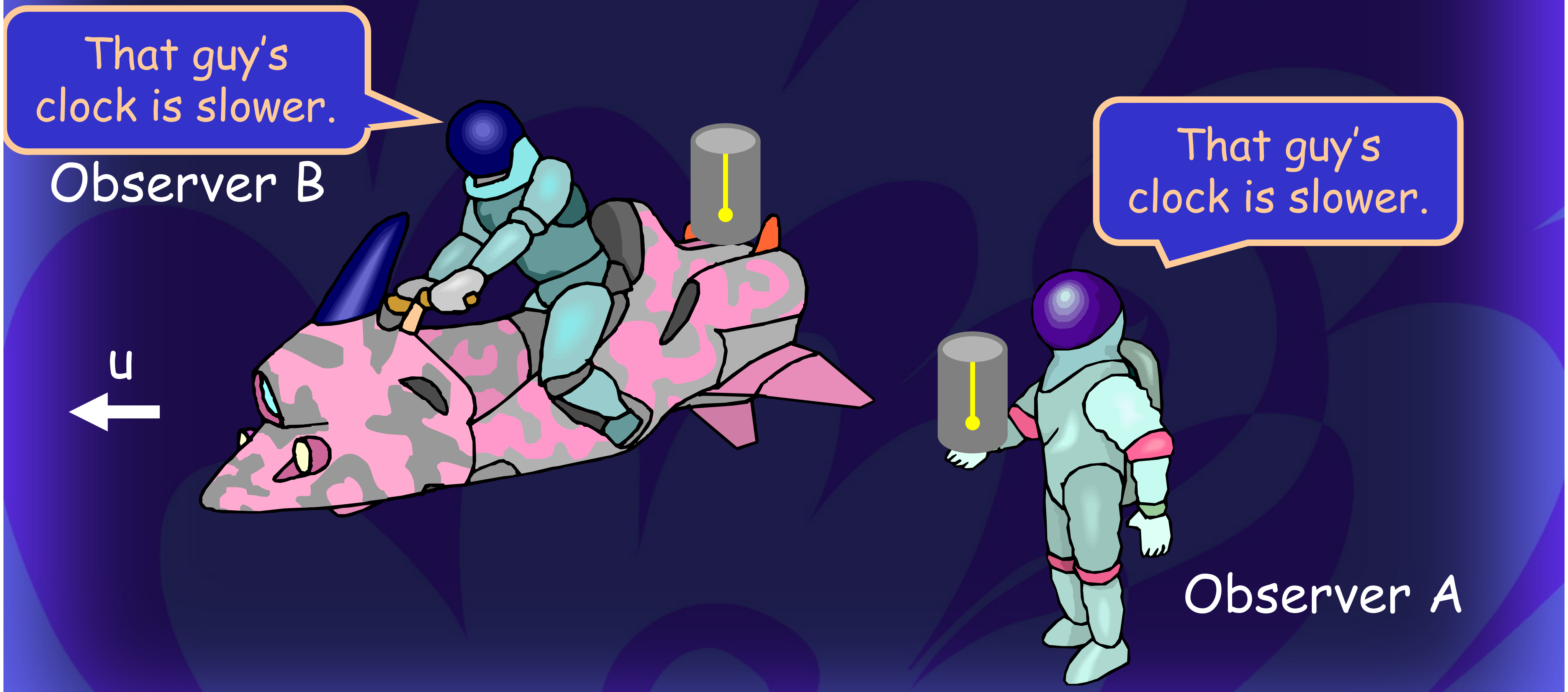


After thinking for a while you will probably conclude that the above statement does not make sense because uniform motion is *relative*!

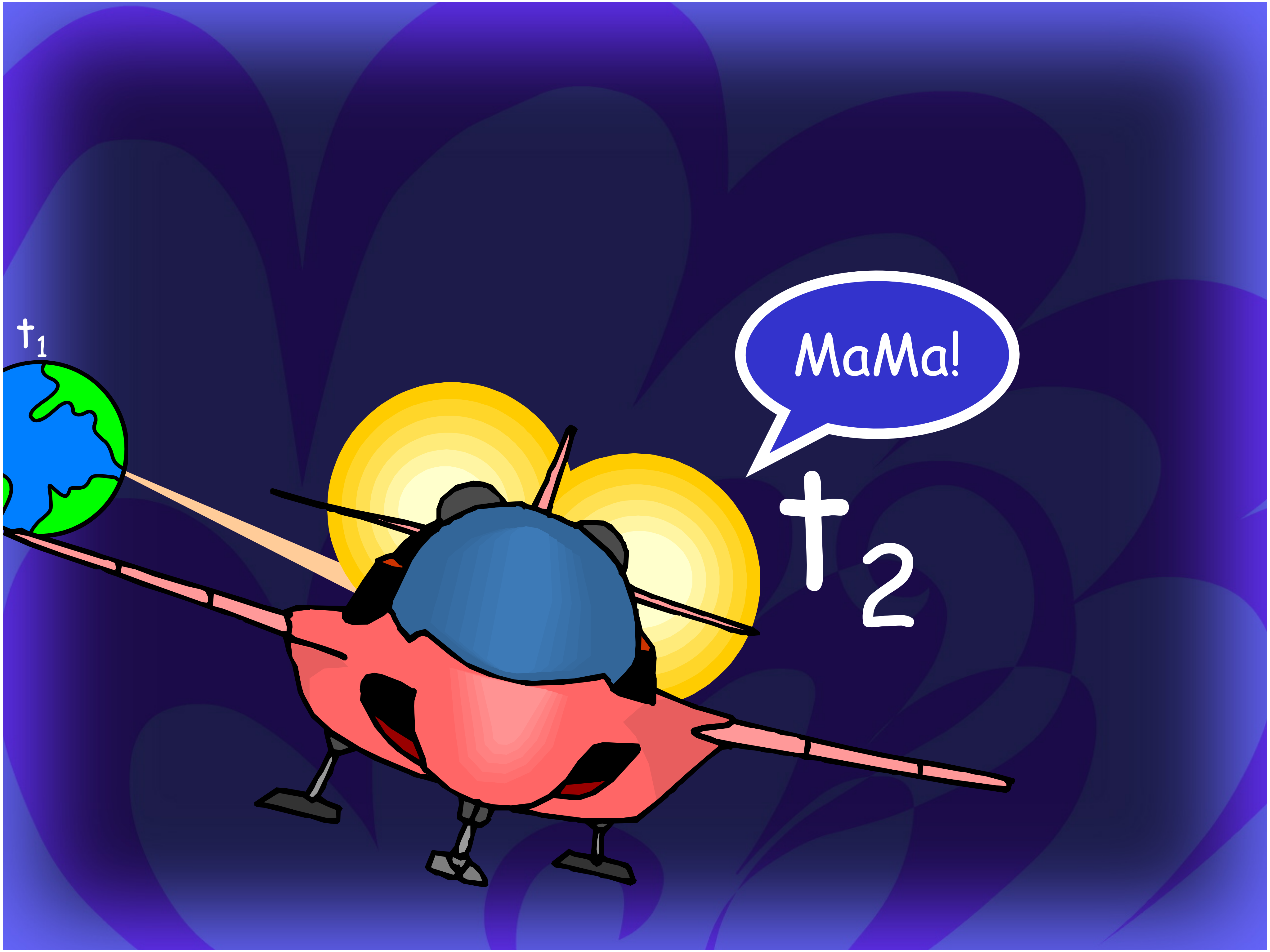
To observer B, he is at rest while A is moving, so he will also see A's clock moving slower

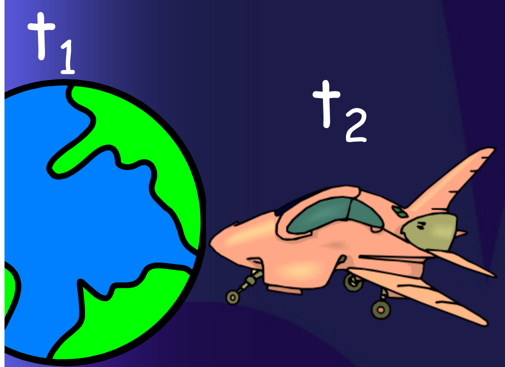


It seems that the two observers will have opposite answer to this question! Who is correct?



This objection is usually rephrased in the so-called **twin paradox**. Suppose there is a pair of twins on earth. The first one, t_1, remains on earth, while the second one, t_2, is sent off in a rocket ship on a trip to a distant star.


t_1
MaMa!
t_2



30 years later…

(according to the clock on earth)

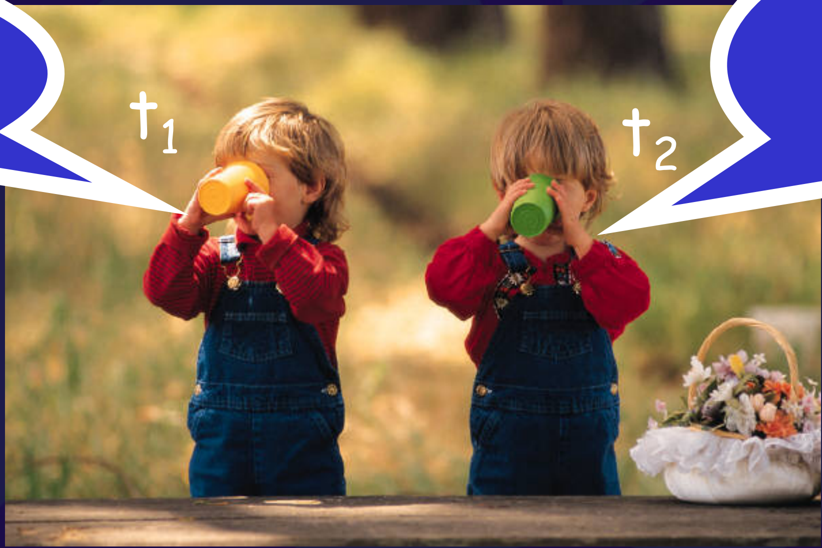
The question is,

is t_1 older than t_2?

The reverse?

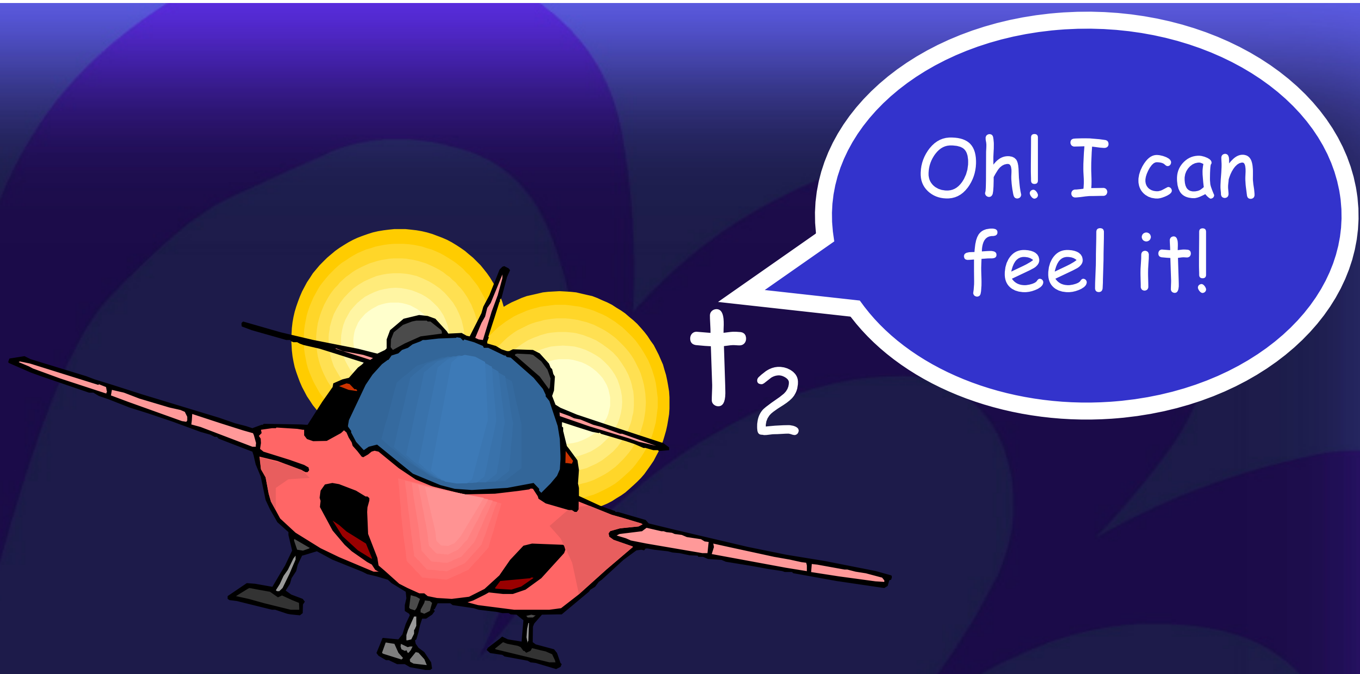
Or do they have the same age?

You are moving. You are younger

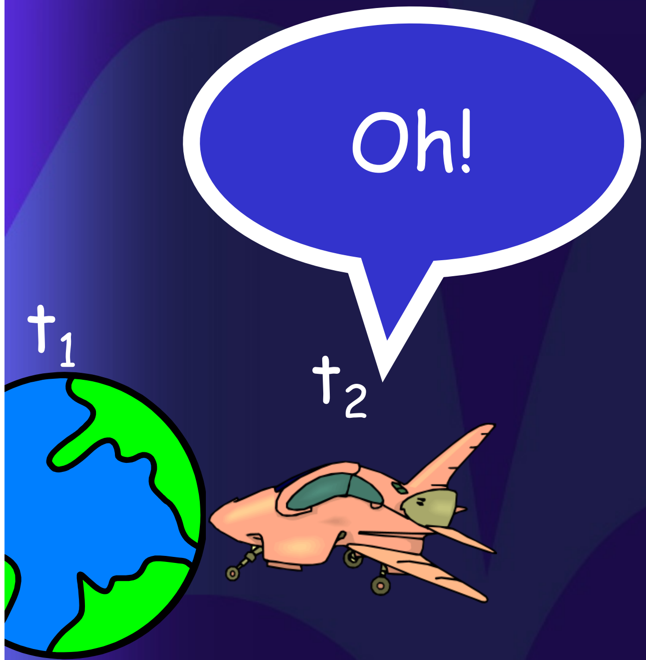


No. I see you are moving. You are younger.

According to t_1, time runs slower in t_2 since t_2 is moving with respect to earth. Therefore, t_2 is younger. However, motion is relative, and according to t_2, time runs slower in t_1 since the earth is moving with respect to the rocket! Who is correct?



The answer to this question lies on the observation that only motions with uniform velocities are relative. Motions that involve acceleration are "absolute". A person cannot detect uniform motion, but he/she can tell whether he/she is accelerating or not!



The motion of the two twins is not totally relative to one another. In order for t_2 to come back, he has to *decelerate* to change direction when returning and also when landing on earth.

Our above analysis is applicable only in an inertial reference frame, i.e. only applicable to t_1. Therefore, the conclusion of t_1 that t_2 is younger should be the correct one. What t_2 should observe is that although the clock of t_1 seems to run slower when he is in uniform motion, he will find that when he is accelerating/decelerating, the clock of t_1 suddenly run much, much faster. The net result is that t_1 is older when t_2 returns to earth.

Classwork

37.5 • The negative pion (π^-) is an unstable particle with an average lifetime of 2.60×10^{-8} s (measured in the rest frame of the pion). (a) If the pion is made to travel at very high speed relative to a laboratory, its average lifetime is measured in the laboratory to be 4.20×10^{-7} s. Calculate the speed of the pion expressed as a fraction of c. (b) What distance, measured in the laboratory, does the pion travel during its average lifetime?

37.11 •• **Why Are We Bombarded by Muons?** Muons are unstable subatomic particles that decay to electrons with a mean lifetime of 2.2 μs. They are produced when cosmic rays bombard the upper atmosphere about 10 km above the earth's surface, and they travel very close to the speed of light. The problem we want to address is why we see any of them at the earth's surface. (a) What is the greatest distance a muon could travel during its 2.2-μs lifetime? (b) According to your answer in part (a), it would seem that muons could never make it to the ground. But the 2.2-μs lifetime is measured in the frame of the muon, and muons are moving very fast. At a speed of $0.999c$, what is the mean lifetime of a muon as measured by an observer at rest on the earth? How far would the muon travel in this time? Does this result explain why we find muons in cosmic rays? (c) From the point of view of the muon, it still lives for only 2.2 μs, so how does it make it to the ground? What is the thickness of the 10 km of atmosphere through which the muon must travel, as measured by the muon? Is it now clear how the muon is able to reach the ground?

37.5. **(a) IDENTIFY and SET UP:** $\Delta t_0 = 2.60\times10^{-8}$ s; $\Delta t = 4.20\times10^{-7}$ s. In the lab frame the pion is created and decays at different points, so this time is not the proper time.

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ says $1-\dfrac{u^2}{c^2} = \left(\dfrac{\Delta t_0}{\Delta t}\right)^2$

$$\frac{u}{c} = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{2.60\times10^{-8}\ \text{s}}{4.20\times10^{-7}\ \text{s}}\right)^2} = 0.998;\ u = 0.998c$$

EVALUATE: $u < c$, as it must be, but u/c is close to unity and the time dilation effects are large.

(b) IDENTIFY and SET UP: The speed in the laboratory frame is $u = 0.998c$; the time measured in this frame is Δt, so the distance as measured in this frame is $d = u\Delta t$.

EXECUTE: $d = (0.998)(2.998\times10^{8}\ \text{m/s})(4.20\times10^{-7}\ \text{s}) = 126\ \text{m}$

37.11. **IDENTIFY and SET UP:** The $2.2\ \mu\text{s}$ lifetime is Δt_0 and the observer on earth measures Δt. The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is l and l_0 is $10\ \text{km}$.

EXECUTE: **(a)** The greatest speed the muon can have is c, so the greatest distance it can travel in $2.2\times10^{-6}\ \text{s}$ is $d = vt = (3.00\times10^{8}\ \text{m/s})(2.2\times10^{-6}\ \text{s}) = 660\ \text{m} = 0.66\ \text{km}$.

(b) $$\Delta t = \frac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \frac{2.2\times10^{-6}\ \text{s}}{\sqrt{1-(0.999)^2}} = 4.9\times10^{-5}\ \text{s}$$

$d = vt = (0.999)(3.00\times10^{8}\ \text{m/s})(4.9\times10^{-5}\ \text{s}) = 15\ \text{km}$

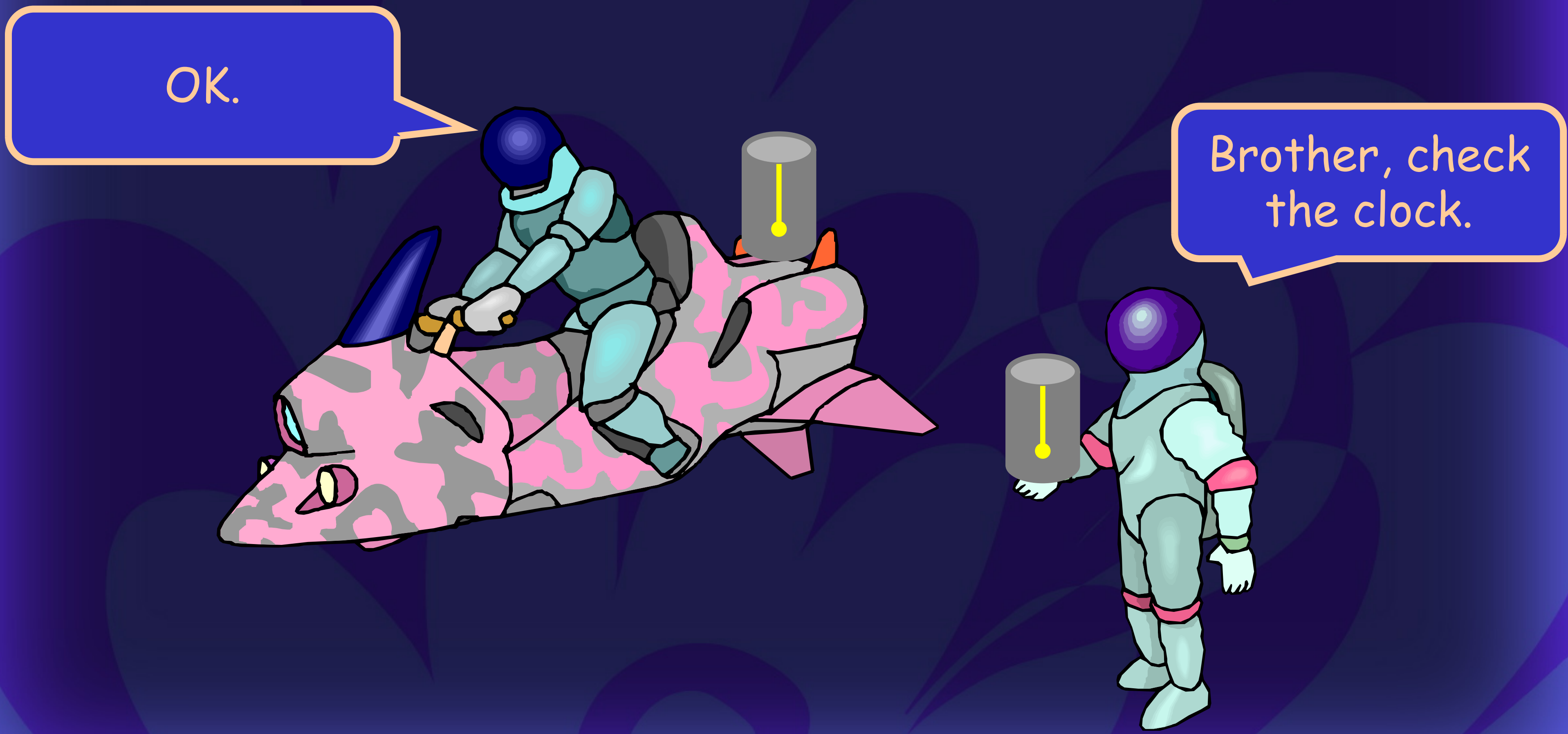
In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c) $l = l_0\sqrt{1-u^2/c^2} = (10\ \text{km})\sqrt{1-(0.999)^2} = 0.45\ \text{km}$

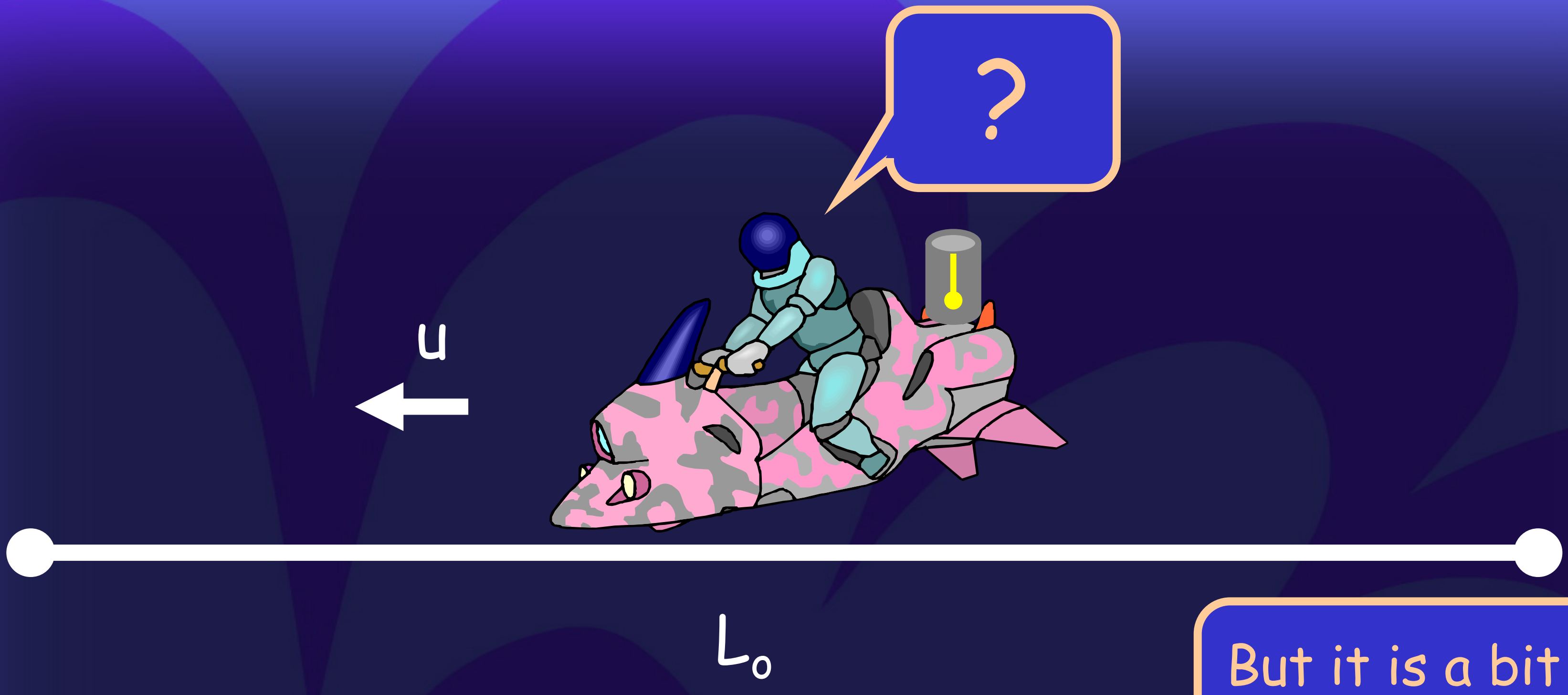
In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

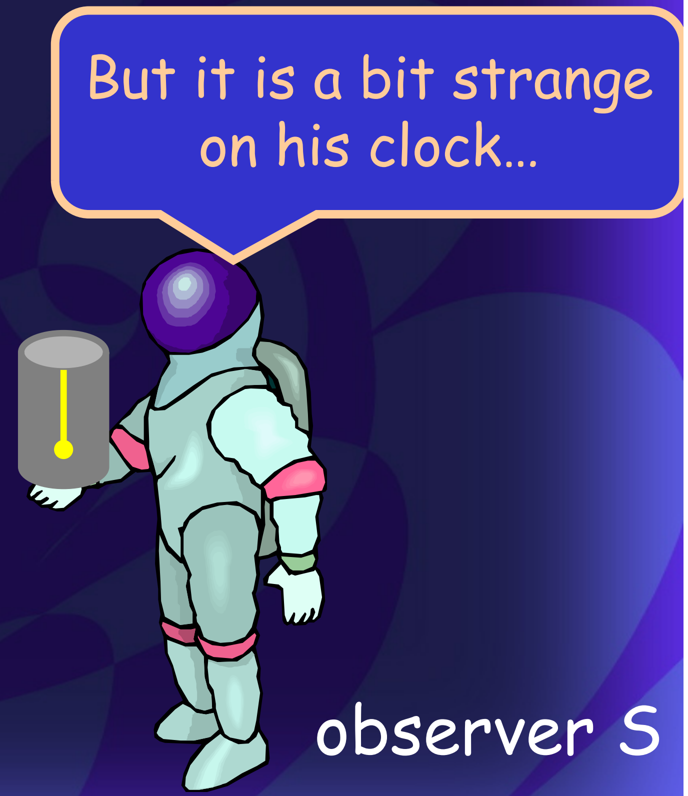
Space contraction

Space contraction is a natural consequence of time dilation if we require consistency in observations by the observers.

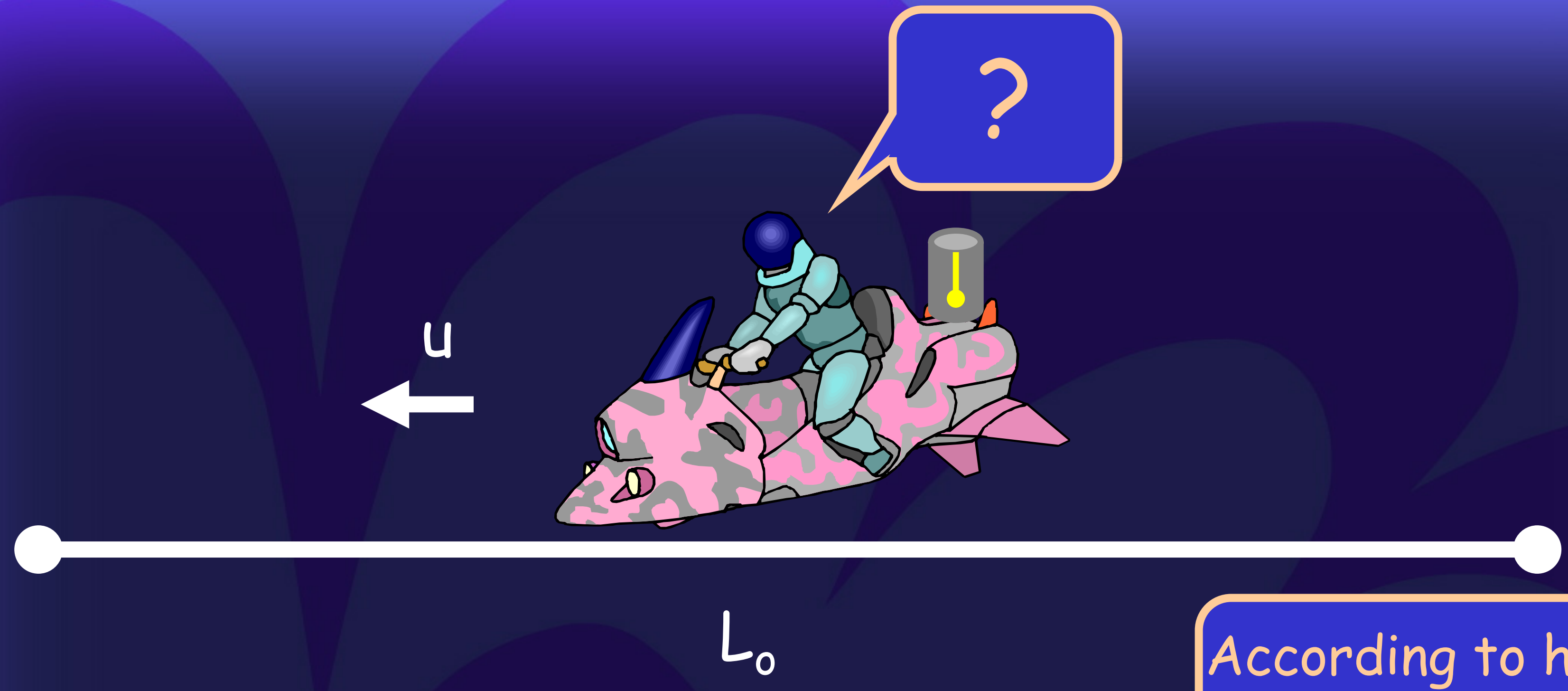


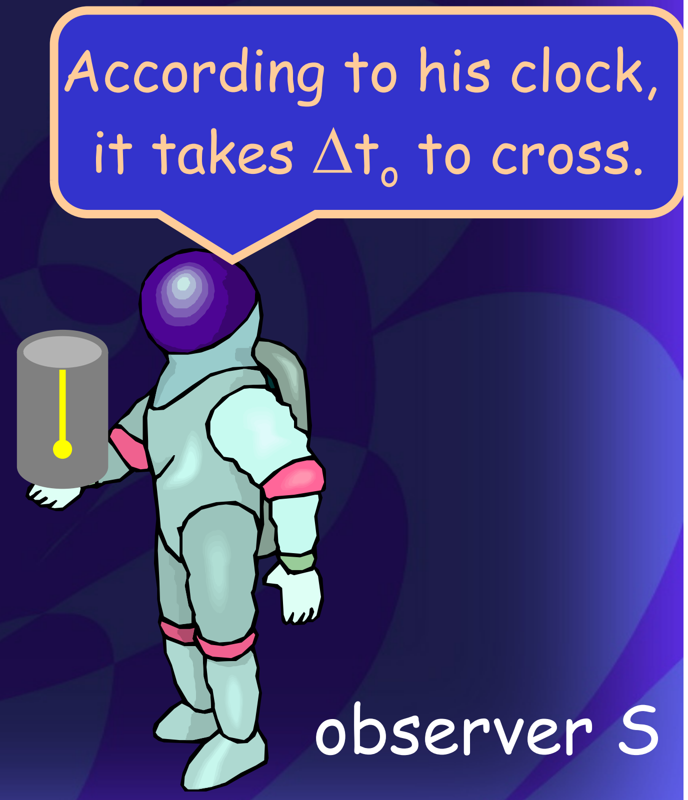
Let's consider a field of length L_o according to an observer S at rest with the field, and a car travelling with velocity u takes time $\Delta t = L_o/u$ to cross the field according to observer S.





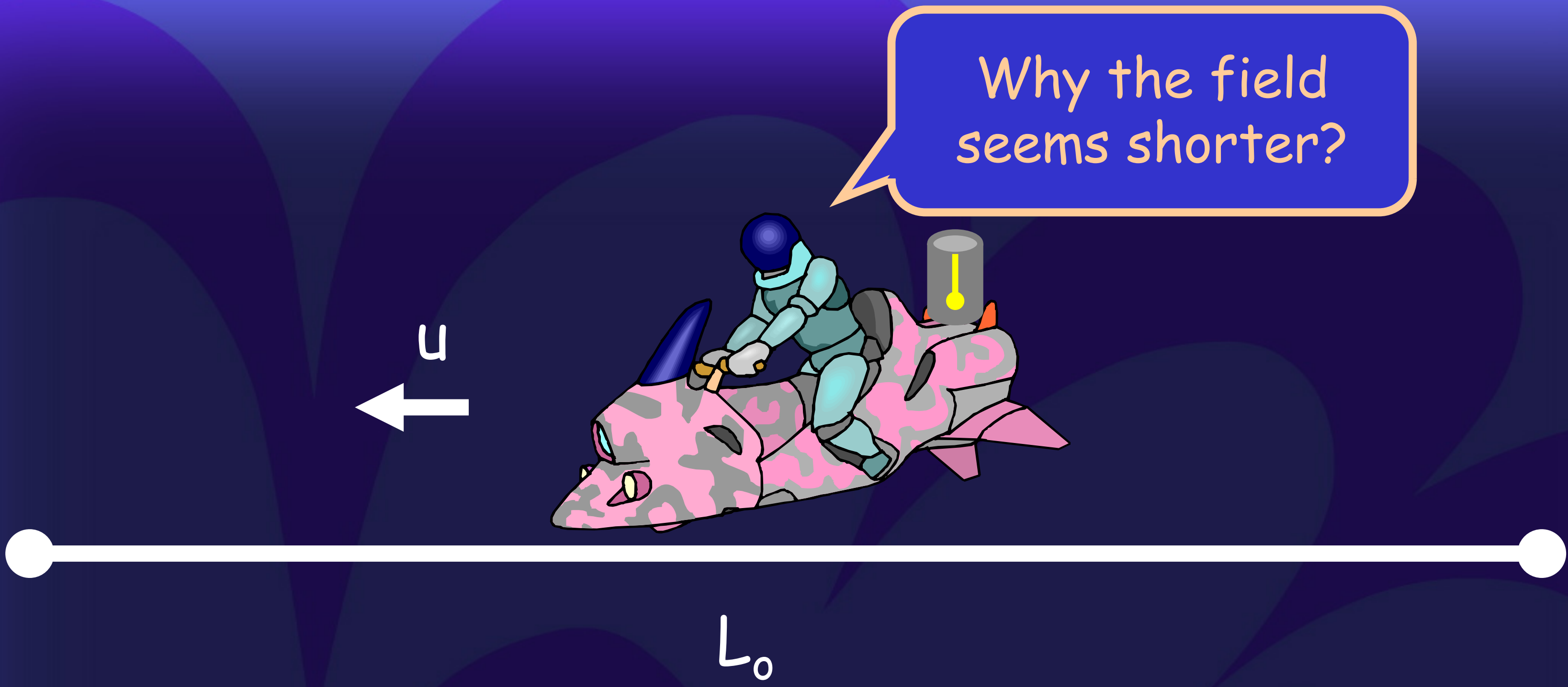
Now let's us consider what happens to a clock inside the car. Well, if you remember time dilation, S will find that her clock runs faster than the clock inside the car.



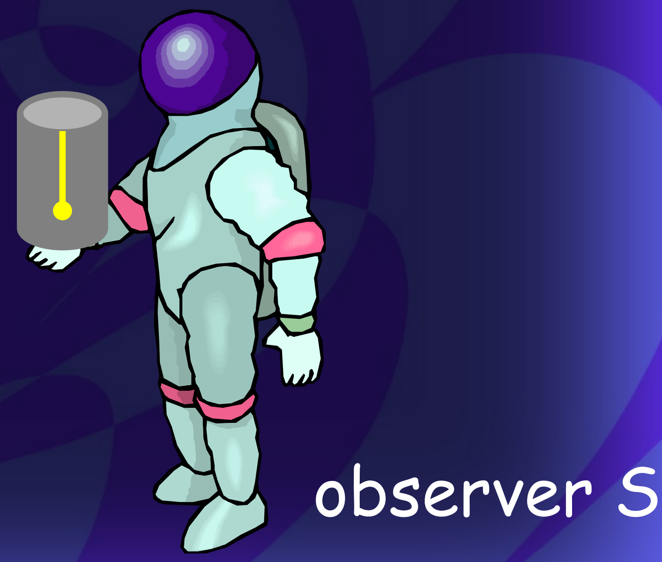


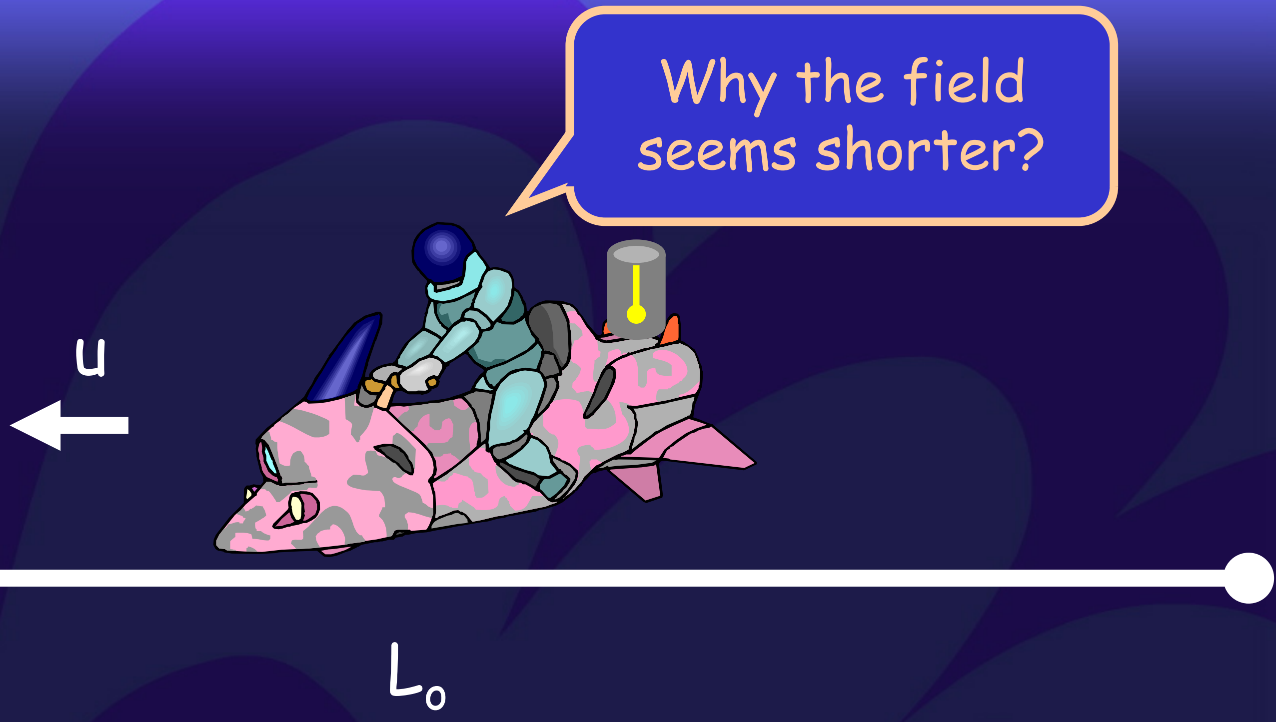
To the clock inside the car, the time Δt_o take to travel through the field is

$$\Delta t_0 = \frac{1}{\gamma}\Delta t = \left(\sqrt{1-\left(\frac{u}{c}\right)^2}\right)\frac{L_0}{u}$$



Since the car is travelling with velocity u with respect to the field, from his point of view, the field is moving towards him at a speed of u





The length of the field he observed will be

$$L = \frac{L_0}{\gamma} = \left(\sqrt{1 - \left(\frac{u}{c}\right)^2}\right) L_0$$

which is shorter than L_0!

observer S

This is called the phenomenon of **space contraction**. To an observer, everything that is travelling with velocity u with respect to him/her will be "shorter" in the direction of motion when compared with their length at rest!

Length contraction and proper length

- A length measured in the frame in which the body is at **rest** (the rest frame of the body) is called a **proper length**.
- Thus l_0 is a proper length in *S′*, and the length measured in any other frame moving relative to *S* is lesser than l_0.
- This effect is called **length contraction**.

Length contraction:

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = \frac{l_0}{\gamma}$$

- Proper length of object (measured in rest frame): l_0
- Speed of second frame relative to rest frame: u
- Lorentz factor relating the two frames: γ
- Length in second frame of reference moving parallel to object's length: l
- Speed of light in vacuum: c

Example 37.4 How long is the spaceship?

A spaceship flies past earth at a speed of $0.990c$. A crew member on board the spaceship measures its length, obtaining the value 400 m. What length do observers measure on earth?

SOLUTION

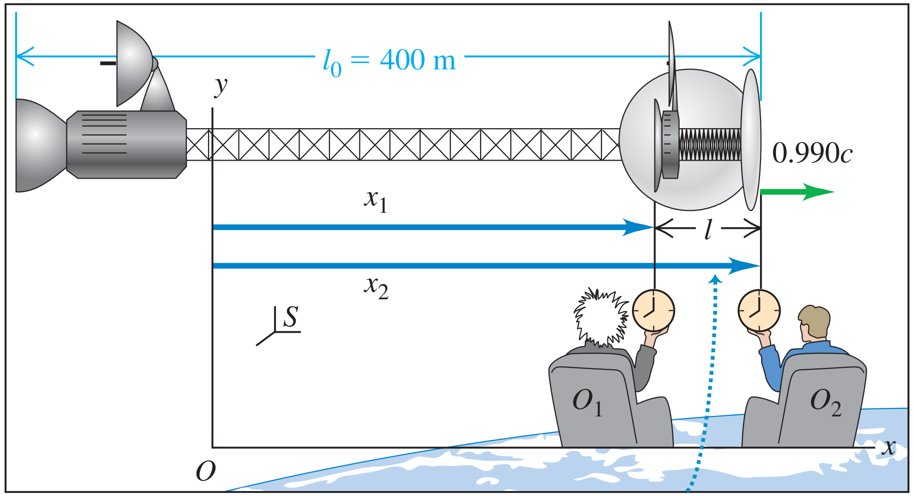
IDENTIFY and SET UP: This problem is about the nose-to-tail length of the spaceship as measured on the spaceship and on earth. This length is along the direction of relative motion (Fig. 37.13), so there will be length contraction. The spaceship's 400-m length is the *proper* length l_0 because it is measured in the frame in which the spaceship is at rest. Our target variable is the length l measured in the earth frame, relative to which the spaceship is moving at $u = 0.990c$.

EXECUTE: From Eq. (37.16), the length in the earth frame is

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = (400\text{ m})\sqrt{1 - (0.990)^2} = 56.4\text{ m}$$

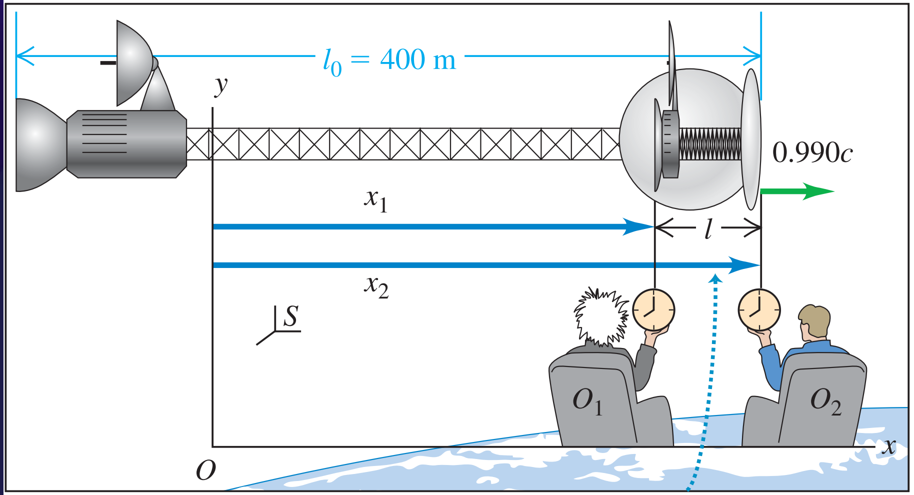
EVALUATE: The spaceship is shorter in a frame in which it is in motion than in a frame in which it is at rest. To measure the length l, two earth observers with synchronized clocks could measure the positions of the two ends of the spaceship simultaneously in the earth's reference frame, as shown in Fig. 37.13. (These two measurements will *not* appear simultaneous to an observer in the spaceship.)

37.13 Measuring the length of a moving spaceship.



The two observers on earth (S) must measure x_2 and x_1 simultaneously to obtain the correct length $l = x_2 - x_1$ in their frame of reference.

37.13 Measuring the length of a moving spaceship.

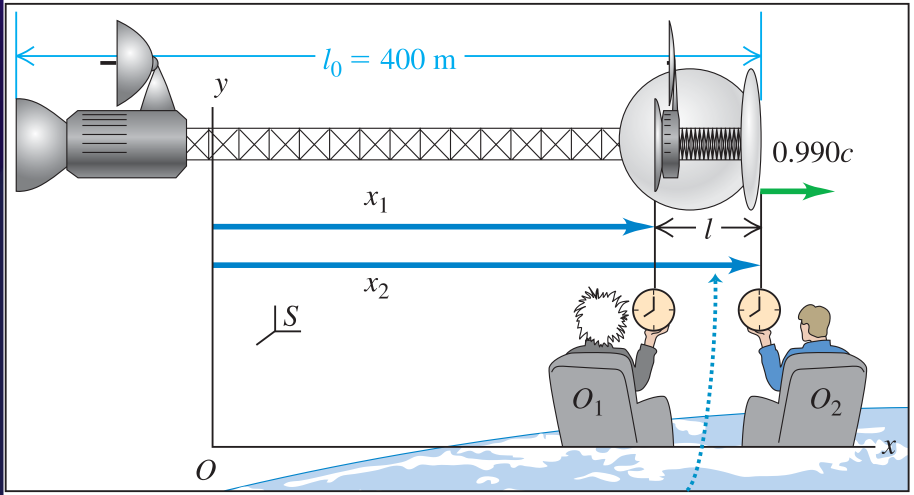


The two observers on earth (S) must measure x_2 and x_1 simultaneously to obtain the correct length $l = x_2 - x_1$ in their frame of reference.

Example 37.5 How far apart are the observers?

Observers O_1 and O_2 in Fig. 37.13 are 56.4 m apart on the earth. How far apart does the spaceship crew measure them to be?

37.13 Measuring the length of a moving spaceship.



The two observers on earth (S) must measure x_2 and x_1 simultaneously to obtain the correct length $l = x_2 - x_1$ in their frame of reference.

Example 37.5 How far apart are the observers?

Observers O_1 and O_2 in Fig. 37.13 are 56.4 m apart on the earth. How far apart does the spaceship crew measure them to be?

SOLUTION

IDENTIFY and SET UP: In this example the 56.4-m distance is the *proper* length l_0. It represents the length of a ruler that extends from O_1 to O_2 and is at rest in the earth frame in which the observers are at rest. Our target variable is the length l of this ruler measured in the spaceship frame, in which the earth and ruler are moving at $u = 0.990c$.

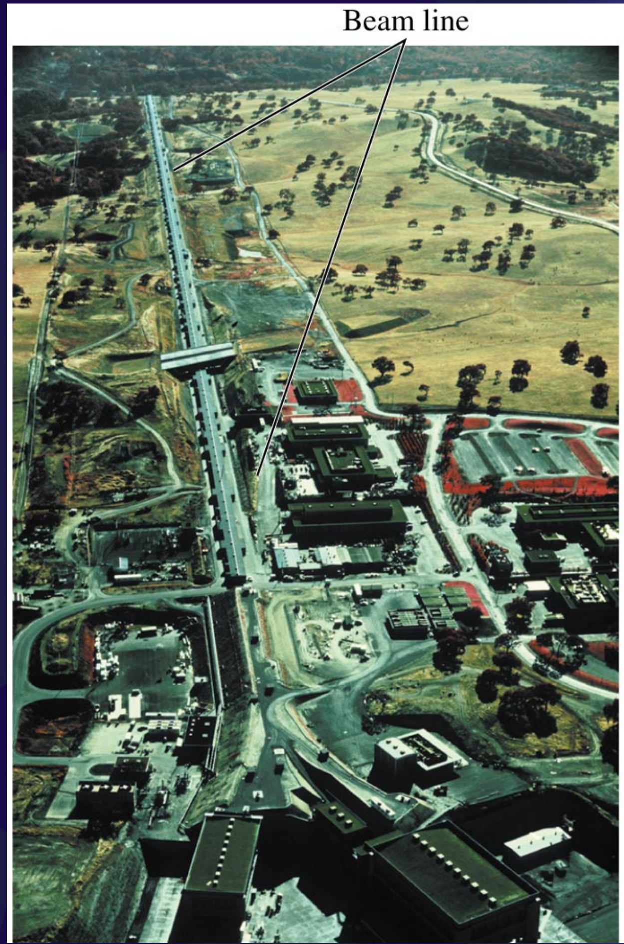
EXECUTE: As in Example 37.4, but with $l_0 = 56.4$ m,

$$l = l_0\sqrt{1 - \frac{u^2}{c^2}} = (56.4\text{ m})\sqrt{1 - (0.990)^2} = 7.96\text{ m}$$

EVALUATE: This answer does *not* say that the crew measures their spaceship to be both 400 m long and 7.96 m long. As measured on earth, the tail of the spacecraft is at the position of O_1 at the same instant that the nose of the spacecraft is at the position of O_2. Hence the length of the spaceship measured on earth equals the 56.4-m distance between O_1 and O_2. But in the spaceship frame O_1 and O_2 are only 7.96 m apart, and the nose (which is 400 m in front of the tail) passes O_2 before the tail passes O_1.

Example of length contraction

- The speed at which electrons traverse the 3-km beam line of the SLAC National Accelerator Laboratory is slower than *c* by less than 1 cm/s.
- As measured in the reference frame of such an electron, the beam line (which extends from the top to the bottom of this photograph) is only about 15 cm long!



Classwork

37.13 • As measured by an observer on the earth, a spacecraft runway on earth has a length of 3600 m. (a) What is the length of the runway as measured by a pilot of a spacecraft flying past at a speed of 4.00×10^7 m/s relative to the earth? (b) An observer on earth measures the time interval from when the spacecraft is directly over one end of the runway until it is directly over the other end. What result does she get? (c) The pilot of the spacecraft measures the time it takes him to travel from one end of the runway to the other end. What value does he get?

37.13. **IDENTIFY:** Apply Eq. (37.16).

SET UP: The proper length l_0 of the runway is its length measured in the earth's frame. The proper time Δt_0 for the time interval for the spacecraft to travel from one end of the runway to the other is the time interval measured in the frame of the spacecraft.

EXECUTE: **(a)** $l_0 = 3600\text{ m}$.

$$l = l_0\sqrt{1-\frac{u^2}{c^2}} = (3600\text{ m})\sqrt{1-\frac{(4.00\times10^7\text{ m/s})^2}{(3.00\times10^8\text{ m/s})^2}} = (3600\text{ m})(0.991) = 3568\text{ m}.$$

(b) $\Delta t = \dfrac{l_0}{u} = \dfrac{3600\text{ m}}{4.00\times10^7\text{ m/s}} = 9.00\times10^{-5}\text{ s}.$

(c) $\Delta t_0 = \dfrac{l}{u} = \dfrac{3568\text{ m}}{4.00\times10^7\text{ m/s}} = 8.92\times10^{-5}\text{ s}.$

EVALUATE: $\dfrac{1}{\gamma} = 0.991$, so Eq. (37.8) gives $\Delta t = \dfrac{8.92\times10^{-5}\text{s}}{0.991} = 9.00\times10^{-5}\text{ s}$. The result from length contraction is consistent with the result from time dilation.