

# DP Mathematics (Level 1)

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The Hong Kong University of Science and Technology



**Course introduction**

September 9, 2023

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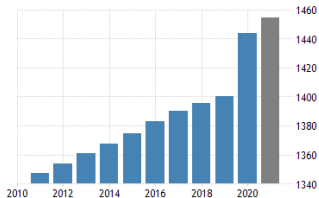
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We will study behaviors of different functions in various aspects.

# Limits and continuity of functions

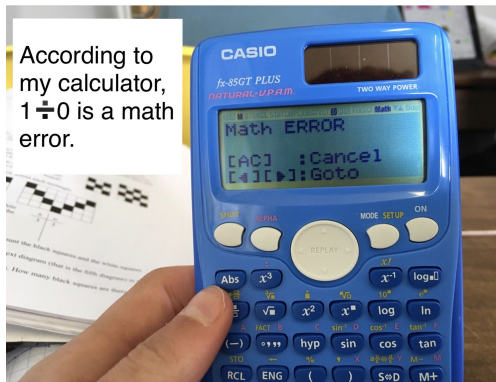
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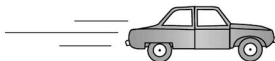
What is actually happening here?

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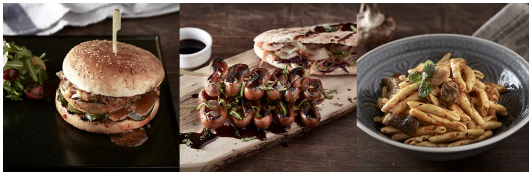
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- Without using a calculator, how do you find a solution to a **complicated equation** such as

$$x = \cos x?$$



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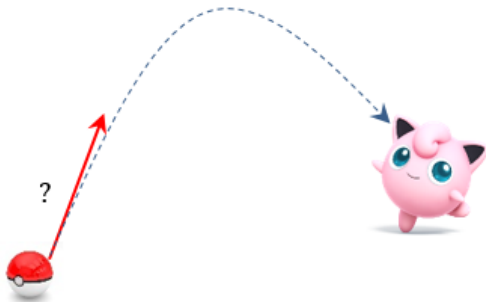
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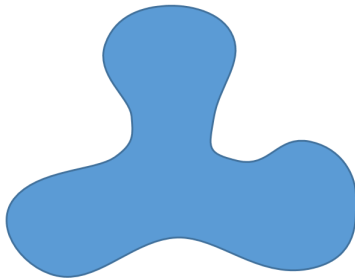
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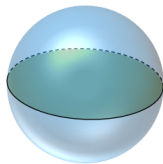
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- Why is the **area** of a circular disk of radius  $r$  given by

$$A = \pi r^2?$$

Why is the **volume** of a spherical ball of radius  $r$  given by

$$V = \frac{4\pi}{3}r^3?$$

How do we compute the **volume**, **surface area**, **length**, etc. of other geometric shapes?



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- Our **numerical computations are easy** – a calculator will not be too helpful usually.
- Some **proof-based problems** will be included.

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The main purpose of this test to better allocate students to a Mathematics course (**Level 1** vs. **Pre-stage**) (**English** vs. **Cantonese**) that is **more suitable** for them.

- You do not need to study or prepare particularly for this test.
- Usage of calculators will not be allowed (and will not be necessary).